Finding Local Maxima, Minima and Inflexion Points

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Local Maxima and Minima

Local maxima & minima (<u>turning</u> <u>points</u>) <u>can</u> occur (but might not occur) where:

1. y' = 0, which is called a **<u>stationary</u> <u>point</u>**.
2. y' is undefined
   e.g. at $x = 0$ for $y = x^{2/3}$ (a **cusp**)
   The IB exam ignores this case, except that you should know that the function is not differentiable there.
3. An endpoint of the domain.
   e.g. at $x = 0$ and 2 for $y = 3x + 2$, $0 \le x \le 2$.

Setting $y' = 0$ locates **<u>local</u>** maxima & minima as opposed to **<u>global</u>** maxima & minima. There can be several local maxima or minima, but there can only be one global maxima or minima. Global max/min are often at $\pm\infty$, which setting $y' = 0$ will not find.
The IB tests for global max/min only by asking for limits, for example $\lim_{x \to 0} \frac{1}{x^2} = +\infty$.

To find the local maxima & minima use any of:

<u>Analytically (To "justify")</u>

<u>1st Derivative Test</u>

Construct a sign table for y'. On each side of points where $y' = 0$ and where y is undefined, check whether y' is positive or negative.

To visualize this draw this as an upward or downward sloping line on the sign table.

1. If y' changes from + to – at that point, there is a local maximum there.
2. If y' changes from – to + at that point, there is a local minimum there.
3. If y' does not change sign, at that point there is a horizontal inflexion point there.

<u>2nd Derivative Test</u>

At the x values where $y' = 0$, check whether y'' is positive or negative.

1. If $y'' < 0$ at that point, there is a local maximum there.
2. If $y'' > 0$ at that point, there is a local minimum there.

<u>Graphically</u>

Sketch the graph of y. Locate min/max visually. Use CALC minimum or maximum to find them.

Be aware that this may not give an exact answer if one is required.

"My calculator said so", does not get the "justify" point.

Inflexion Points

<u>Inflexion</u> <u>points</u> are points on the curve where the concavity changes. The concavity <u>can</u> change (but might not change) where:

1. $y'' = 0$.
2. y'' is undefined.
   For example at $x = 1$ for $y = x^{5/3} + 5x^{2/3}$. $x = 1$ is an inflexion point.
   For $f(x) = \frac{1}{(x-1)^2}$, The concavity changes at $x = 1$, but $x = 1$ is **<u>not</u>** an inflexion point, because $f(x)$ is not defined there. The IB exam ignores this case.

To find the inflexion points use any of:

<u>Analytically (To "justify") use the 2nd Derivative Test</u>

Check y'' on each side of the x values where $y'' = 0$.
Construct a sign table for y'' by testing convenient values of x.
If y'' changes sign on opposite sides of the x value where $y'' = 0$:

1. If $f'(x) = 0$ there, it is a horizontal inflexion point.
2. If $f'(x) \neq 0$ there, it is a normal inflexion point.

To visualize this if $y'' > 0$ draw a concave up curve and if $y'' < 0$ draw a concave down curve.

<u>Graphically</u>

Graph y in Y1 in Y=. Turn it off by clicking on the =" sign.
Graph y' (**<u>not</u>** y) using ALPHA F2 3: nDeriv(and ALPHA F4
Set xRes = 3 (in WINDOW) to speed it up.

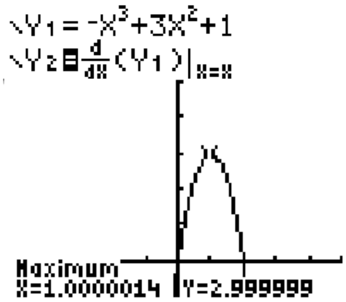
Use CALC minimum or maximum to find the local maxima & minima of y'.

Either a local maxima or minima of y' indicates an inflexion point.

Be aware that this will not give an exact answer if one is required.

Example

Find the inflexion point of $y = -x^3 + 3x + 1$



Since y' has a maximum at $x = 1$, y has an inflexion point at $x = 1$

"My calculator said so", does not get the "justify" point.