Finding Local Maxima, Minima and Inflexion Points for AA SL

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Maxima and Minima

Maxima & minima† (which are called **turning points**) occur at points where $\boldsymbol{y}' = 0$ (which are called **stationary points)**.

A point where $y' = 0$ could be a maximum, a minimum or a horizontal inflection point.

Sometimes the question will say something like "find the *x*-coordinate of the maximum".
In this case set $\boldsymbol{y}' = 0$, solve for x and you are done.

Sometimes you are required to "justify" whether the point where $y'=0$ is a maximum, a minimum or a horizontal inflection point.

To "justify" that a point is a maximum & a minimum you can do either of the 1st Derivative Test or the 2nd Derivative Test. There is no need to do both.

1st Derivative Test

Construct a sign table for y'. I.e. on each side of the x values where $y' = 0$, use convenient values of x (e.g. $x = 0$) to check whether y' is positive or negative.

To visualize this
when $y' > 0$ draw an upward sloping line on the sign table and
when $y' < 0$ draw a downward sloping line on the sign table.

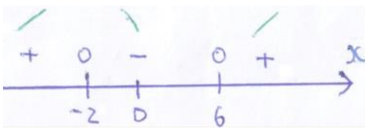
1. If y' changes from + to – where $y' = 0$, there is a local maximum there.
2. If y' changes from – to + where $y' = 0$, there is a local minimum there.
3. If y' does not change sign where $y' = 0$, there is a horizontal inflexion point there.

Example

$y = x^3 – 6x^2 – 36x + 24$
$y' = 3x^2 – 12x – 36 = 0$
$x^2 – 4x – 12 = 0$
$(x – 6)(x + 2) = 0$
$x = 6$ or -2
Test $y'(x)$ at $x = 0$ (Use 0 because $y(0)$ is easy to calculate) . $y'(0) = -36 < 0$.
So put a minus sign in the region $-2 < x < 6$ and plus signs in the other regions*.

Draw upward slanting lines in the + regions and downward sloping lines in the – regions.

So the point where $x = -2$ is a maximum and the point where $x = 6$ is a minimum.



* Because there are no repeated roots

2nd Derivative Test

At the x values where $y' = 0$, check whether y'' is positive or negative.

1. If $y'' < 0$ at such a point, there is a local maximum there.
2. If $y'' > 0$ at such a point, there is a local minimum there.

Example (continued from the above example)

$y = x^3 – 6x^2 – 36x + 24$
$y' = 3x^2 – 12x – 36$
$y'' = 6x – 12$

We already know that our two candidate points are $x = 6$ or -2.

$y''(-2) = -12 – 12 = -24 < 0$. Therefore the point where $x = -2$ is a maximum.
$y''(6) = 36 – 12 = 24 > 0$. Therefore the point where $x = 6$ is a minimum.

† Where I say maxima & minima above I am referring to “local” maxima & minima as opposed to “global” maxima & minima, which occur at vertical asymptotes and often when $x = \pm\infty$. Global maxima & minima are not in the AA SL syllabus.

Inflexion Points

<u>Inflexion</u> <u>points</u> are points on the curve where the concavity changes.
The concavity <u>usually</u> changes (but might not change) where: $y'' = 0$.

To find the inflexion points:

Construct a sign table for y'' by testing convenient values of x.
If y'' changes sign on opposite sides of the x value where $y'' = 0$ and:
If $y'(x) = 0$ there, it is a horizontal inflexion point.
If $y'(x) \neq 0$ there, it is a normal inflexion point.

Example (continued from the above example)

$y'' = 6x – 12 = 0$
$x = 2$
Checking to the left of $x = 2$, $y''(1) = 6 – 12 = -6 < 0$ so concave down.
Checking to the right of $x = 2$, $y''(3) = 18 – 12 = 6 > 0$ so concave up.
Since the concavity changes at $x = 2$, $x = 2$ is an inflection point.