How to sketch $f'(x)$ from the graph of $f(x)$ and vice versa

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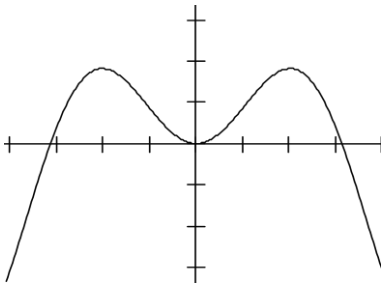
1. Where $f(x)$ has a minimum or a maximum (i.e. its "instantaneous slope" is ***zero***), $f'(x)$ equals ***zero***. Put an "x" on the x-axis of $f'(x)$ wherever $f(x)$ has a minimum or a maximum.

2. If you can guess the apparent functional form of $f(x)$, the power of the derivative is one less than the power of the function. This is because $\frac{d(x^n)}{dx} = nx^{n-1}$.

Example if $f(x)$ looks like a cubic, $f'(x)$ will look like a parabola.

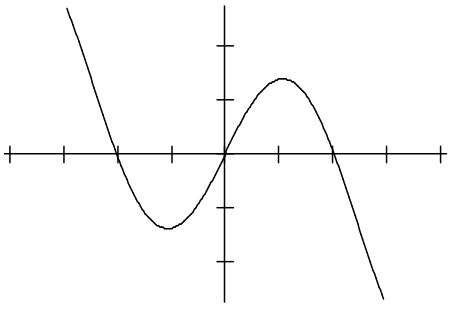
3. The derivative is the "instantaneous slope". So if in interval (a, b) the **<u>slope</u> of $f(x)$** is negative, then in interval (a, b) the **<u>value</u> (<u>NOT</u> the slope) of $f'(x)$** will be negative. And vice versa.

Example: If $f(x)$ looks like:



1. Since there are maximums at x = ±2 and a minimum at x = 0, $f'(x)$ will have zeros there.
2. $f(x)$ looks like a quartic, so $f'(x)$ should look like a cubic.
3. Since the **slope** of $f(x)$ above is positive in intervals $(\infty, -2)$ and $(0, 2)$, $f'(x)$ will be positive in those intervals (and negative elsewhere). So $f'(x)$ should look something like:

So $f'(x)$ looks like:



4. To sketch $f(x)$ from a graph of $f'(x)$ you run the above advice backwards.