Inverse Functions

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An inverse function "undoes" whatever its corresponding function does.

$f^{-1}(x)$ should be **pronounced** "*f* inverse of *x*", not *f* minus 1.

Careful, the symbol is confusing: $f^{-1}(x) \neq (f(x))^{-1} = \dfrac{1}{f(x)}$ = the **reciprocal** of $f(x)$.

$f(2) = 3$ means that when $x = 2$, $y = 3$; $f^{-1}(3) = 2$ means that when $y = 3$, $x = 2$.

Examples:

$f(x)$ = Function	$f^{-1}(x)$ = Inverse Function*
x^3	$\sqrt[3]{x}$
$x + 2$	$x - 2$
$3x$	$\frac{x}{3}$
10^x	$\log x$
$2x+3$	$\frac{x-3}{2}$

*One could just as well reverse any pair in the above two columns, e.g. listing $\log x$ as the function & 10^x as the inverse function.

Finding the Inverse of $f(x)$

1. Replace $f(x)$ by y.
2. Interchange x & y.
3. Solve for y.
4. Replace y by $f^{-1}(x)$.

Definition of the Inverse Function

Algebraically the definition of the inverse function is: $f^{-1}(f(x)) = x$ or $f(f^{-1}(x)) = x$.

Graphically the definition of the inverse function is that the function and its inverse are reflections of each other about the line $y = x$ and that the graphical properties of x and y are exchanged (see below).

The graphical properties of x and y are exchanged:

For $f(x)$ & $f^{-1}(x)$:

1. The y-intercept of one is the x-intercept of the other.
2. The domain of one is the range of the other.
3. The vertical asymptote of one is the horizontal asymptote of the other.
4. If their graphs cross, they must cross on the line $y = x$.

Horizontal Line Test

A function can have an inverse **function** only if passes the Horizontal Line Test (HLT), which is that if any **horizontal** line intersects the graph of $f(x)$ in more than one point, then $f(x)$ fails the HLT.

Remember that the Vertical Line Test (VLT) determines whether a relation is a *function*.