How to Draw Reciprocal Functions

Dr. William J. Larson - http://MathsTutorGeneva.ch/

If $f(x) = \dfrac{1}{g(x)} = \left(g(x)\right)^{-1}$, then f(x) and g(x) are reciprocal functions. Note that $g(x) = \dfrac{1}{f(x)}$.

Reciprocals are <u>**not**</u> inverses, i.e. $(f(x))^{-1} \neq f^{-1}(x)$.

Example If $f(x) = x^2$ and $g(x) = \dfrac{1}{x^2}$, $g(x)$ & $g(x)$ are reciprocal functions.

1. Where $g(x)$ has a **zero**, $f(x)$ has a **vertical asymptote**,
   because if $g(x)$ approaches 0, $f(x)$ approaches "1/0" i.e. ∞.
   And vice versa, i.e., where $g(x)$ has a **vertical asymptote**, $f(x)$ has a **zero**,
   because if g(x) approaches ∞, $f(x)$ approaches "1/∞" i.e. 0.
2. Calculate and **plot any minima or maxima**.

   Example: If $g(x)$ has a maximum at (3, 2), then $f(x)$ will have a minimum at $(3, \frac{1}{2})$.
3. If $g(x)$ goes through 1, f(x) also **goes through 1**. Plot the point.
   Example: If $g(x)$ goes through the point (4, 1), then $f(x)$ will also go through the point (4, 1).
4. **Horizontal Asymptotes**
   a) If $g(x)$ has a **horizontal asymptote** at y = a, then $f(x)$ will have a horizontal asymptote at y = 1/a.
   b) If $g(x)$ has a **horizontal asymptote at y = 0** (the x-axis), $f(x)$ will not have a horizontal asymptote.
   c) If $g(x)$ **does not have a horizontal asymptote**, $f(x)$ will have a horizontal asymptote at y = 0 (the x-axis).
5. If $g(x) > 0$, then $f(x) > 0$, because 1 divided by a **positive** number is a positive number.
   If $g(x) < 0$, then $f(x) < 0$, because 1 divided by a **negative** number is a negative number.
6. Draw a curve through the points plotted, respecting the above rules.

A function and its reciprocal	Another function and its reciprocal