Integration by Parts

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The formula for integration by parts is $\int u\, dv = uv - \int v\, du$. It is in the formula booklet.

Integration by parts is needed when you need to integrate the product of two functions. Typical examples are: $\int xe^{-x}dx$ or $\int x^2\cos(x)dx$ or $\int x\ln(x)dx$.

LIPET

$\int u\, dv = uv - \int v\, du$

But what should you make u & what dv?

Choose *u* as whichever function comes first in this list:

L: Logarithmic functions: $\ln x$, $\log_2(x)$, etc.
I: Inverse trigonometric functions: $\arctan x$, $\arcsin x$, etc.
P: Polynomial functions: x, x^2, $\sqrt{x}$ etc.
E: Exponential functions: e^x, 13^x, etc.
T: Trigonometric functions: $\sin x$, $\tan x$, etc.

Then make d*v* the other function. You can remember the list by the mnemonic LIPET.

(Another common mnemonic is ILATE.)

$\int \ln(x)dx$, $\int \arcsin(x)dx$, $\int \arctan(x)dx$, etc. are done with parts.

let $u = \ln(x)$, $\arcsin(x)$ or $\arctan(x)$ and $dv = dx$.

Tabular Repeated Integration by Parts

After integration by parts some expressions, for example, $\int x^2 e^{-x}\ dx$ and $\int x^2\ cosx\ dx$ require a second application of integration by parts. Repeated integration by parts is in the IB syllabus.

Repeated integration by parts requires a lot of tedious bookkeeping. But instead of explicitly listing out each of the steps, you can create a table with three columns, for example for $\int x^2 e^{-x}\ dx$, listing x^2 and each of its successive *derivatives* in the first column and e^{-x} and each of its successive *integrals* in the second column. In the third column write +, –, +, etc.

You then diagonally multiply the first entry in the first column by the second entry in the second column and by the third sign in the third column. Then multiply the second entry in the first column by the third entry in the second column and by the fourth sign in the third column and continue this process. That sounds confusing, but it's not too bad. It is less confusing than the normal method. See the example below.

Once there is a derivative of 0 in the first column, it does not need to be multiplied by anything.

Then add each of these terms and the constant C.

Example $\int x^2 e^{-x}\ dx$

$+\,x^2$	e^{-x}	+
$+\,2x$	$-e^{-x}$	–
$+\,2$	e^{-x}	+
0	$-e^{-x}$	–
		+

Adding the *diagonal* products: $\int x^2 e^{-x}\ dx = -x^2 e^{-x} - 2x\, e^{-x} - 2e^{-x} + C$

$\int e^x \cos x\, dx$ by parts

Although $\int e^x\ cosx\ dx$ requires repeated integration by parts, a different trick is needed. The trick for $\int e^x\ cosx\ dx$ is that after double integration by parts we get:
$\int e^x\ cosx\ dx = e^x\ cosx + e^x \sin x - \int e^x\ cosx\ dx$.

Note that we still have an integral on the right side, but the same integral!
This gives: $\int e^x\ cosx\ dx = \frac{1}{2}(e^x\ cosx + e^x \sin x)$