The Properties of Logarithms

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The definition of logarithms: $y = \log_a x \Leftrightarrow a^y = x \quad (a > 0, a \neq 1)$

This means that $y = \log_a x$ and $y = a^x$ are inverses.

$\log_b x$ is **pronounced "log base b of x"**

The Natural Logarithm = $\ln x \equiv \log_e x$

ln x is pronounced "el en x". ln stands for **n**atural **l**ogarithm. $e \cong 2.718281828459\ldots$

The Common Logarithm: $\log x \equiv \log_{10} x$

Elementary Properties of Logarithms

$\log_a x$ laws	Because	log x laws	because	ln x laws	because
$\log_a 1 = 0$	$a^0 = 1$	$\log 1 = 0$	$10^0 = 1$	$\ln 1 = 0$	$e^0 = 1$
$\log_a a = 1$	$a^1 = a$	$\log 10 = 1$	$10^1 = 10$	$\ln e = 1$	$e^1 = e$
$\log_a a^x = x$	a^x & $\log_a x$ are inverses	$\log 10^x = x$	10^x & log x are inverses	$\ln e^x = x$	e^x & ln x are inverses
$a^{\log_a x} = x$	a^x & $\log_a x$ are inverses	$10^{\log x} = x$	10^x & log x are inverses	$e^{\ln x} = x$	e^x & ln x are inverses

The Basic Laws of Logarithms

Law	because
1. $\log_a(u\,v) = \log_a u + \log_a v$	$a^u a^v = a^{u+v}$
2. $\log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v$	$\frac{a^u}{a^v} = a^{u-v}$
3. $\log_a u^x = x \log_a u$	$(a^x)^n = a^{xn}$

More rules of Logarithms - in descending order of usefulness!

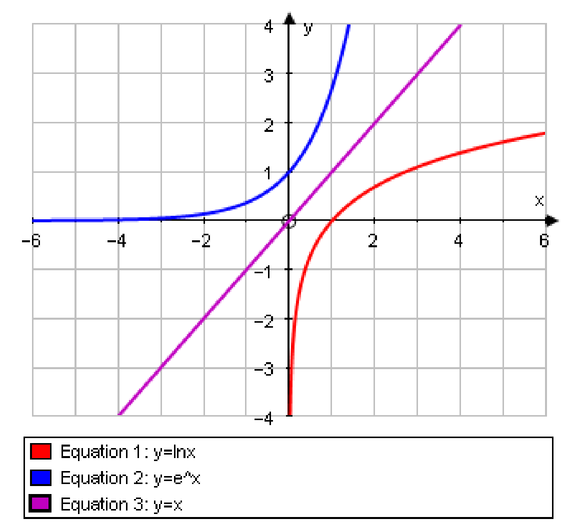
1. <u>**The Change of Base Formula**</u>

   $\log_a c = \frac{\log_b c}{\log_b a}$, for any b, $b > 0, b \neq 1$

2. $\log_{a^y} a^x = \frac{x}{y}$
3. $\log_a b = \frac{1}{\log_b a}$
4. $a^x = e^{x \ln a}$
5. $\frac{\log_b c}{\log_b a} = \frac{\log_d c}{\log_d a}$
6. $a^{\log_b c} = c^{\log_b a}$
7. $\log_a x = \frac{\ln x}{\ln a} = \frac{\log x}{\log a}$
8. $\log_{a^b} c^d = \frac{d}{b}\log_a c = \log_{a^{1/d}} c^{1/b}$
9. $\log_{\frac{1}{a}} x = -\log_a x$

Solving Equations with Logarithms & Exponents

1. $\log_a f(x) = \log_a g(x) \Leftrightarrow f(x) = g(x)$, $a > 0, a \neq 1$
2. $a^{f(x)} = a^{g(x)} \Leftrightarrow f(x) = g(x)$, $a > 0, a \neq 1$
3. $f(x)^a = g(x)^a \Leftrightarrow f(x) = g(x)$ If a is an **even** integer $\neq 0$, $f(x) = -g(x)$ is also a solution.



Log functions and exponential functions are inverses.
So they are reflections of each other across the line y = x.

	$y = \log_b x$	$y = b^x$
asymptote	vertical: x = 0	horizontal: y = 0
axis intercept	x = 1	y = 1