Asymptotes of Rational Functions and Logs

Dr William J. Larson - MathsTutorGeneva.ch

An **asymptote** is a line that the graph of an equation approaches, but never reaches.

A **rational function** is a polynomial divided by another polynomial. (The ratio of two polynomials)

Horizontal Asymptotes of Rational Functions

A horizontal asymptote is a horizontal line that the function approaches as x approaches +∞ or -∞.

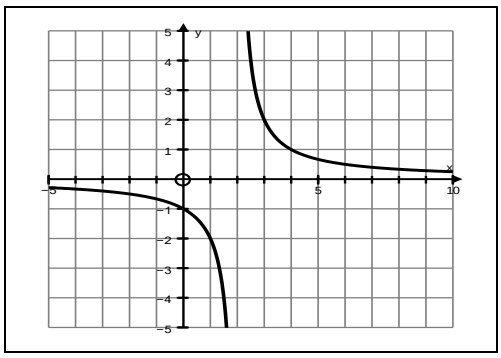
To investigate horizontal asymptotes, the only thing you need to consider is the highest powers in the numerator and denominator respectively.

There are 3 cases:

1. If the power in the denominator is bigger, then the x-axis ($y = 0$) is a horizontal asymptote.

Example $f(x) = \dfrac{2}{x+3}$.

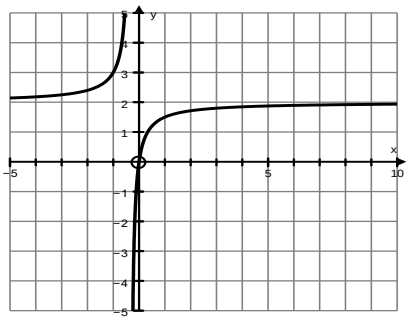
The line y = 0 is a horizontal asymptote.



2. If the powers are the same, there is a horizontal asymptote at y equals the ratio of the leading coefficients (the coefficients of the highest power terms) in the numerator and denominator.

Example $f(x) = \dfrac{6x}{3x+1}$. HA: y = 2

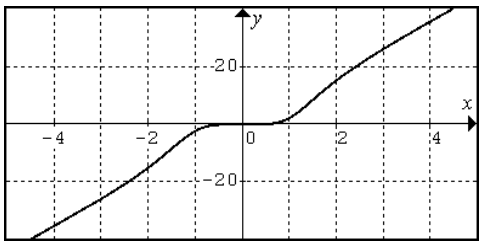
The line $y = \dfrac{6}{3}$, i.e. y = 2 is a horizontal asymptote.



Notice that $f(x) = \dfrac{x-1}{x+2}$ (on the left) has a horizontal asymptote at y = 1.

3. If the power in the numerator is bigger, there are no horizontal asymptotes.

Example $f(x) = \dfrac{9x^5 - 1}{x^4 + 3}$.



A further complication can be tested.

Example $f(x) = \dfrac{2}{x+3} + 4$.

The line $y = 0$ is a horizontal asymptote of $f(x) = \dfrac{2}{x+3}$; but the "+4" shifts everything up by 4. So the horizontal asymptote is $y = 4$.

The graph of a function <u>can</u> cross a horizontal asymptote.

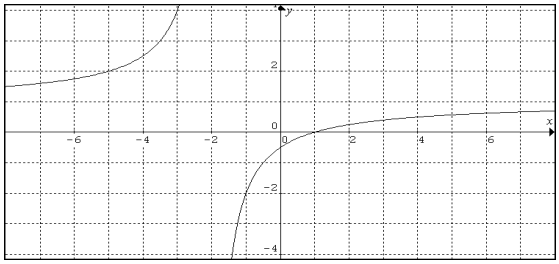
Vertical Asymptotes of Rational Functions

There is a vertical asymptote at those values of x that make the denominator equal to zero.

Example $f(x) = \dfrac{x-1}{x+2}$.

There is a vertical asymptote if x + 2 = 0.

Therefore the line x = -2 is a vertical asymptote.



The graph of a function <u>cannot</u> cross a vertical asymptote, because if it did it would not be a function.

Vertical Asymptotes of Logarithms

A logarithm has a vertical asymptote of where it's argument is zero.

Example $f(x) = \log(x-1)$

There is a vertical asymptote at $x - 1 = 0$, therefore at $x = 1$.