Trigonometry Facts

Dr. William J. Larson - MathsTutorGeneva.ch

θ	sin θ	Memory Trick for sin θ count 0, 1, 2, 3, 4	cos θ (Same as sin θ, but in reverse order)	tan $\theta = \frac{\sin\theta}{\cos\theta}$
$0, 2\pi$	0	$\frac{\sqrt{0}}{2} = 0$	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	$\frac{\sqrt{4}}{2} = 1$	0	undefined
π	0		-1	0
$\frac{3\pi}{2}$	-1		0	undefined

Arc length = $s = r\,\theta$ (**<u>in radians only</u>**)

Area of a sector = $\frac{1}{2}r^2\theta = \frac{1}{2}sr$ (**<u>in radians only</u>**)

Area of a triangle = $\frac{1}{2}ab\sin C$

(plus 2 more formulas interchanging the letters)

Trig functions definitions

Function	Using the sides of a right triangle	Using a point (x, y) on the terminal side	Using the point (x, y) on the unit circle
sin θ	$\frac{opp}{hyp}$	$\frac{y}{r}$	y
cos θ	$\frac{adj}{hyp}$	$\frac{x}{r}$	x
tan θ	$\frac{opp}{adj}$	$\frac{y}{x}$	$\frac{y}{x}$

$r^2 = x^2 + y^2$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

The quadrants in which the function is positive:
Mnemonic: "**A**ll **S**tudents **T**ake **C**alculus"

S (sine)	A (all)
T (tangent)	C (cosine)

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \times \tan B}$$

$\sin 2\theta = 2 \sin\theta \cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$a^2 = b^2 + c^2 - 2bc\cos A$ (plus 2 more interchanging the letters)

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ (plus 2 more interchanging the letters)

sin(-θ) = -sin θ
cos(-θ) = +cos θ
tan(-θ) = -tan θ

The co-functions of complementary angles are equal:

sin θ = cos $(90 - \theta)$ *cos θ = sin* $(90 - \theta)$

tan θ = cot $(90 - \theta)$ *cot θ = tan* $(90 - \theta)$

sec θ = csc $(90 - \theta)$ *csc θ = sec* $(90 - \theta)$

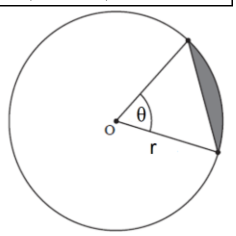
$\sin\theta = \sin(n\pi - \theta)$, n odd, $\cos(\theta) = \cos(n\pi - \theta)$, n even;

$\tan(3\pi - x) = -\tan x$, $\sin(\pi + x) = -\sin x$

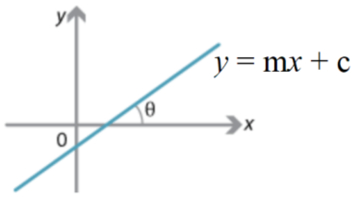
$\sin(180° - \theta) = \sin\theta$	$\sin(\theta + 90°) = +\cos\theta$	$\sin(\theta + 180°) = -\sin\theta$
$\cos(180° - \theta) = -\cos\theta$	$\cos(\theta + 90°) = -\sin\theta$	$\cos(\theta + 180°) = -\cos\theta$
$\tan(180° - \theta) = -\tan\theta$	$\tan(\theta + 90°) = -\cot\theta$	$\tan(\theta + 180°) = +\tan\theta$

The area of the shaded region is

$\frac{1}{2}r^2(\theta - \sin\theta)$ with θ in radians.



tan $\theta = m$



The angle between two lines with slopes

m_1 and m_2: $\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$

$y = a\sin(b(x - c)) + d$, $|a|$ = amplitude, $\frac{2\pi}{b}$ = period,

c = horizontal shift ($c > 0$ is a shift right),
d = vertical shift ($d > 0$ is a shift up)