The Vector Equation of a Line in for AI HL

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Vector Form of the equation of a Line

$\vec{r} = \vec{a} + \lambda\vec{b}$

or

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \lambda \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix},$$

where $\vec{a}$ is the position vector giving the position of a point on the line at $\lambda = 0$,
$\vec{b}$ is a vector parallel to the line (the direction vector of the line) and parameter λ is a variable.
(x_0, y_0, z_0) is a point on the line.
Two lines are parallel if their direction vectors are scalar multiples of each other.
In other words $\vec{r}_1 = \vec{p}_1 + \lambda\vec{d}_1$ and $\vec{r}_2 = \vec{p}_2 + \mu\vec{d}_2$ are parallel, if $\vec{d}_1 = k\,\vec{d}_2$ $\ for\ \ some\ \ scalar\ \ k$.

Example

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix}$$

Parametric Form of the equation of a Line

$x(\lambda) = x_0 + x_1\,\lambda$
$y(\lambda) = y_0 + y_1\,\lambda$
$z(\lambda) = z_0 + z_1\,\lambda$
Where parameter λ is a variable and x_0, y_0, z_0, x_1, y_1 and z_1 are scalar constants.
(x_0, y_0, z_0) is a point on the line.

$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ is the direction vector of the line.

Example

$x(t) = 3 + 4\lambda$
$y(t) = 5 + 6\lambda$
$z(t) = -2 + \lambda$

Converting Between the Forms

This is easy since it just requires adding or removing vector brackets.

Example: Convert
$x = 4\lambda + 3$,
$y = 6\lambda + 5$,
$z = \lambda - 2$
to parametric vector form.

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix}$$