Parallel and Perpendicular Vectors

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To prove that two non-zero vectors $\vec{A}$ and $\vec{B}$ are parallel,

Show $\vec{A} = k\vec{B}$, where k is a scalar**.**

Example

$$\vec{a} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix}, \quad b = \begin{pmatrix} -9 \\ 6 \\ -15 \end{pmatrix}; \quad \vec{a} \text{ is parallel to } \vec{b}, \text{ because } \vec{b} = -3\vec{a}$$

To prove that two non-zero vectors $\vec{A}$ and $\vec{B}$ are perpendicular:

Show $\vec{A} \bullet \vec{B} = 0$

Example

$$\vec{a} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}; \quad \vec{a} \text{ is perpendicular to } \vec{b}, \text{ because } \vec{a} \bullet \vec{b} = 6 - 6 + 0 = 0$$

To construct a vector $\vec{C}$ perpendicular to non-zero vectors $\vec{A}$ and $\vec{B}$:

$\vec{C} = \vec{A} \times \vec{B}$.

Example

$$\vec{C} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix} \times \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -15 \\ 10 \\ 13 \end{pmatrix}$$

Checking:

$$\begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix} \bullet \begin{pmatrix} -15 \\ 10 \\ 13 \end{pmatrix} = -45 - 20 + 65 = 0$$

$$\begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} -15 \\ 10 \\ 13 \end{pmatrix} = -30 + 30 + 0 = 0$$