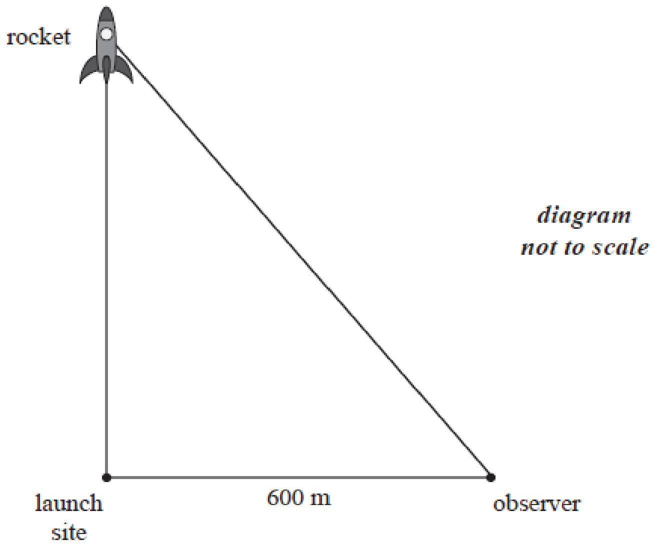
Related Rates 2008-2014 w MS

2. *[6 marks]*

A rocket is rising vertically at a speed of 300m s^{-1} when it is 800 m directly above the launch site. Calculate the rate of change of the distance between the rocket and an observer, who is 600 m from the launch site and on the same horizontal level as the launch site.



3. *[7 marks]*

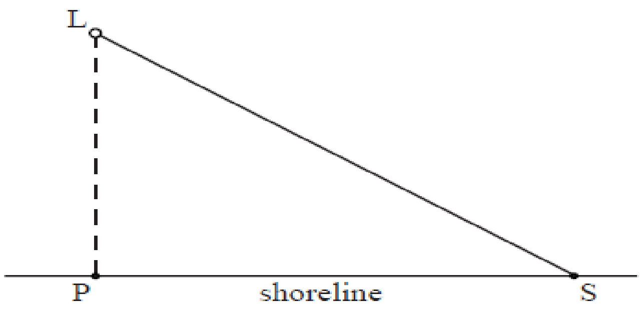
A ladder of length 10 m on horizontal ground rests against a vertical wall. The bottom of the ladder is moved away from the wall at a constant speed of 0.5m s^{-1}. Calculate the speed of descent of the top of the ladder when the bottom of the ladder is 4 m away from the wall.

4. *[6 marks]*

Paint is poured into a tray where it forms a circular pool with a uniform thickness of 0.5 cm. If the paint is poured at a constant rate of $4\text{c m}^3\text{s}^{-1}$, find the rate of increase of the radius of the circle when the radius is 20 cm.

5. *[8 marks]*

A lighthouse L is located offshore, 500 metres from the nearest point P on a long straight shoreline. The narrow beam of light from the lighthouse rotates at a constant rate of 8π radians per minute, producing an illuminated spot S that moves along the shoreline. You may assume that the height of the lighthouse can be ignored and that the beam of light lies in the horizontal plane defined by sea level.



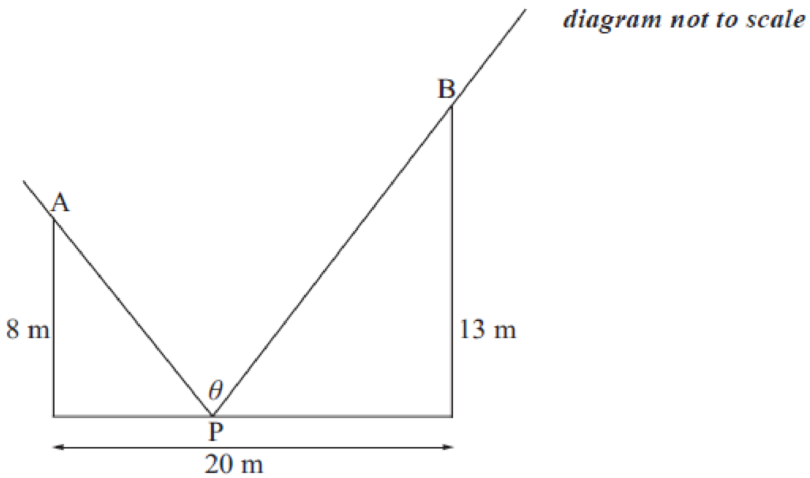
When S is 2000 metres from P,

(a) show that the speed of S, correct to three significant figures, is $214\,000$ metres per minute;

(b) find the acceleration of S.

6a. *[2 marks]*

A straight street of width 20 metres is bounded on its parallel sides by two vertical walls, one of height 13 metres, the other of height 8 metres. The intensity of light at point P at ground level on the street is proportional to the angle θ where $\theta = \text{A}\hat{\text{P}}\text{B}$, as shown in the diagram.



Find an expression for θ in terms of x, where x is the distance of P from the base of the wall of height 8 m.

6b. *[2 marks]*

(i) Calculate the value of θ when $x = 0$.

(ii) Calculate the value of θ when $x = 20$.

6c. *[2 marks]*

Sketch the graph of θ, for $0 \leqslant x \leqslant 20$.

6d. *[6 marks]*

Show that $\frac{d\theta}{dx} = \frac{5(744-64x-x^2)}{(x^2+64)(x^2-40x+569)}$.

6e. *[3 marks]*

Using the result in part (d), or otherwise, determine the value of x corresponding to the maximum light intensity at P. Give your answer to four significant figures.

6f. *[4 marks]*

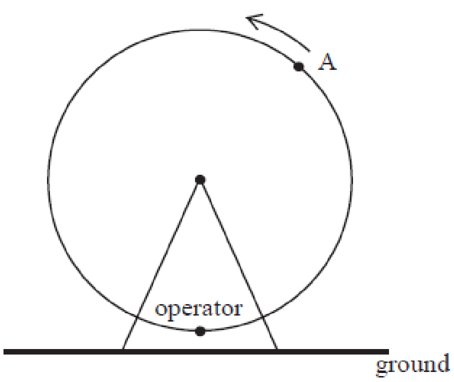
The point P moves across the street with speed 0.5m s^{-1}. Determine the rate of change of θ with respect to time when P is at the midpoint of the street.

7. *[6 marks]*

A stalactite has the shape of a circular cone. Its height is 200 mm and is increasing at a rate of 3 mm per century. Its base radius is 40 mm and is decreasing at a rate of 0.5 mm per century. Determine if its volume is increasing or decreasing, and the rate at which the volume is changing.

9. *[10 marks]*

Below is a sketch of a Ferris wheel, an amusement park device carrying passengers around the rim of the wheel.

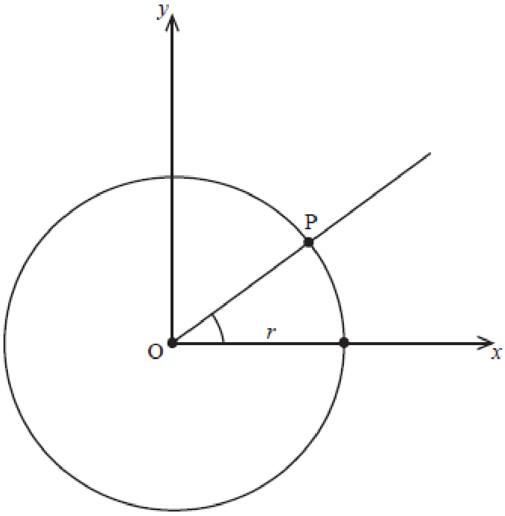


(a) The circular Ferris wheel has a radius of 10 metres and is revolving at a rate of 3 radians per minute. Determine how fast a passenger on the wheel is going vertically upwards when the passenger is at point A, 6 metres higher than the centre of the wheel, and is rising.

(b) The operator of the Ferris wheel stands directly below the centre such that the bottom of the Ferris wheel is level with his eyeline. As he watches the passenger his line of sight makes an angle α with the horizontal. Find the rate of change of α at point A.

10. *[7 marks]*

The diagram below shows a circle with centre at the origin O and radius $r > 0$.



A point P(x , y) , ($x > 0, y > 0$) is moving round the circumference of the circle.

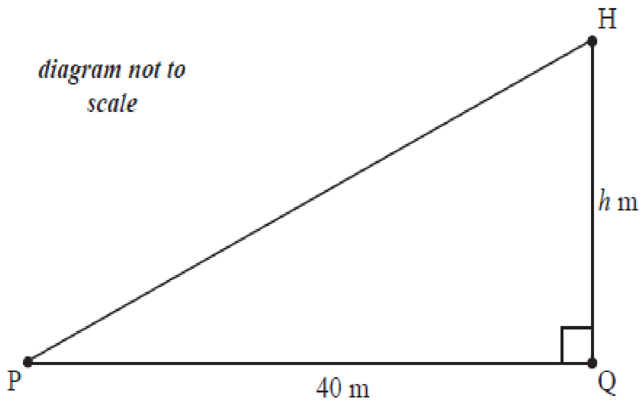
Let $m = \tan\left(\arcsin \frac{y}{r}\right)$.

(a) Given that $\frac{\mathrm{d}y}{\mathrm{d}t} = 0.001r$, show that $\frac{\mathrm{d}m}{\mathrm{d}t} = \left(\frac{r}{10\sqrt{r^2 - y^2}}\right)^3$.

(b) State the geometrical meaning of $\frac{\mathrm{d}m}{\mathrm{d}t}$.

11. *[7 marks]*

A helicopter H is moving vertically upwards with a speed of 10 ms . The helicopter is h m directly above the point Q which is situated on level ground. The helicopter is observed from the point P which is also at ground level and PQ $= 40$ m. This information is represented in the diagram below.



When $h = 30$,

(a) show that the rate of change of $\mathrm{H\hat{P}Q}$ is 0.16 radians per second;

(b) find the rate of change of PH.

13. *[5 marks]*

Sand is being poured to form a cone of height h cm and base radius r cm. The height remains equal to the base radius at all times. The height of the cone is increasing at a rate of 0.5 cm min^{-1}.

Find the rate at which sand is being poured, in $\text{cm}^3 \text{ min}^{-1}$, when the height is 4 cm.

Related Rates 2008-2014 MS

2. *[6 marks]*

Markscheme

let x = distance from observer to rocket

let h = the height of the rocket above the ground

METHOD 1

$\frac{\mathrm{d}h}{\mathrm{d}t} = 300 \text{ when } h = 800$ ***A1***

$x = \sqrt{h^2 + 360\,000} = (h^2 + 360\,000)^{\frac{1}{2}}$ ***M1***

$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{h}{\sqrt{h^2+360\ 000}}$ ***A1***

when h = 800

$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t}$ ***M1***

$= \frac{300h}{\sqrt{h^2+360\ 000}}$ ***A1***

$= 240\ (\mathrm{ms}^{-1})$ ***A1***

[6 marks]

METHOD 2

$h^2 + 600^2 = x^2$ ***M1***

$2h = 2x\,\frac{\mathrm{d}x}{\mathrm{d}h}$ ***A1***

$\frac{\mathrm{d}x}{\mathrm{d}h} = \frac{h}{x}$

$= \frac{800}{1000}\left(= \frac{4}{5}\right)$ ***A1***

$\frac{\mathrm{d}h}{\mathrm{d}t} = 300$ ***A1***

$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t}$ ***M1***

$= \frac{4}{5} \times 300$

$= 240\ (\mathrm{ms}^{-1})$ ***A1***

[6 marks]

METHOD 3

$x^2 = 600^2 + h^2$ ***M1***

$2x\,\frac{\mathrm{d}x}{\mathrm{d}t} = 2h\,\frac{\mathrm{d}h}{\mathrm{d}t}$ ***A1A1***

when h = 800, x =1000

$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{800}{1000} \times \frac{\mathrm{d}h}{\mathrm{d}t}$ ***M1A1***

$= 240\ (\mathrm{ms}^{-1})$ ***A1***

[6 marks]

METHOD 4

Distance between the observer and the rocket $= (600^2 + 800^2)^{\frac{1}{2}} = 1000$ ***M1A1***

Component of the velocity in the line of sight $= \sin\theta \times 300$

(where $\theta =$ angle of elevation) ***M1A1***

$\sin\theta = \frac{800}{1000}$ ***A1***

component $= 240\ (\text{ms}^{-1})$ ***A1***

[6 marks]

Examiners report

Questions of this type are often open to various approaches, but most full solutions require the application of 'related rates of change'. Although most candidates realised this, their success rate was low. This was particularly apparent in approaches involving trigonometric functions. Some candidates assumed constant speed – this gained some small credit. Candidates should be encouraged to state what their symbols stand for.

3. *[7 marks]*

Markscheme

let x, y (m) denote respectively the distance of the bottom of the ladder from the wall and the distance of the top of the ladder from the ground

then,

$x^2 + y^2 = 100$ ***M1A1***

$2x\frac{\mathrm{d}x}{\mathrm{d}t} + 2y\frac{\mathrm{d}y}{\mathrm{d}t} = 0$ ***M1A1***

when $x = 4,\ y = \sqrt{84}$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 0.5$ ***A1***

substituting, $2 \times 4 \times 0.5 + 2\sqrt{84}\frac{\mathrm{d}y}{\mathrm{d}t} = 0$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}t} = -0.218\text{m s}^{-1}$ ***A1***

(speed of descent is 0.218m s^{-1})

[7 marks]

Examiners report

[N/A]

4. *[6 marks]*

Markscheme

$V = 0.5\pi r^2$ ***(A1)***

EITHER

$\frac{dV}{dr} = \pi r$ ***A1***

$\frac{dV}{dt} = 4$ ***(A1)***

applying chain rule ***M1***

for example $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV}$

OR

$\frac{dV}{dt} = \pi r\frac{dr}{dt}$ ***M1A1***

$\frac{dV}{dt} = 4$ ***(A1)***

THEN

$\frac{dr}{dt} = 4 \times \frac{1}{\pi r}$ ***A1***

when $r = 20,\ \frac{dr}{dt} = \frac{4}{20\pi}$ or $\frac{1}{5\pi}$ (cm s^{-1}) ***A1***

Note: Allow *h* instead of 0.5 up until the final ***A1***.

[6 marks]

Examiners report

There was a large variety of methods used in this question, with most candidates choosing to implicitly differentiate the expression for volume in terms of *r*.

5. *[8 marks]*

Markscheme

(a) the distance of the spot from P is $x = 500\tan\theta$ ***A1***

the speed of the spot is

$\frac{\mathrm{d}x}{\mathrm{d}t} = 500\sec^2\theta\,\frac{\mathrm{d}\theta}{\mathrm{d}t}\ \left(= 4000\pi\sec^2\theta\right)$ ***M1A1***

when $x = 2000,\ \sec^2\theta = 17\ (\theta = 1.32581\ldots)\left(\frac{\mathrm{d}\theta}{\mathrm{d}t} = 8\pi\right)$

$\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = 500 \times 17 \times 8\pi$ ***M1A1***

speed is $214\ 000$ (metres per minute) ***AG***

Note: If their displayed answer does not round to $214\ 000$, they lose the final ***A1***.

(b) $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 8000\pi\sec^2\theta\tan\theta\,\frac{\mathrm{d}\theta}{\mathrm{d}t}$ or $500 \times 2\sec^2\theta\tan\theta\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2$ ***M1A1***

$\left(\text{since } \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = 0\right)$

$= 43\,000\,000\ (= 4.30 \times 10^7)\ (\text{metres per minute}^2)$ ***A1***

[8 marks]

Examiners report

This was a wordy question with a clear diagram, requiring candidates to state variables and do some calculus. Very few responded appropriately.

$\Rightarrow t = \frac{2}{3}\sqrt{\frac{5000\pi}{2\pi}} = 33\,\frac{1}{3}$ hours ***A1***

OR

$\int_{5000\pi}^{0} \frac{\mathrm{d}V}{\sqrt{V}} = -3\sqrt{2\pi}\int_0^T \mathrm{d}t$ ***M1A1A1***

Note: Award ***M1*** for attempt to use definite integrals, ***A1*** for correct limits and ***A1*** for correct integrands.

$\left[2\sqrt{V}\right]_{5000\pi}^{0} = 3\sqrt{2\pi}T$ ***A1***

$T = \frac{2}{3}\sqrt{\frac{5000\pi}{2\pi}} = 33\,\frac{1}{3}$ hours ***A1***

[7 marks]

Examiners report

This question was found to be challenging by many candidates and there were very few completely correct solutions. Many candidates did not seem able to find the volume of revolution when taken about the *y*-axis in (a). Candidates did not always recognize that part (b) did not involve related rates. Those candidates who attempted the question made some progress by separating the variables and integrating in (c) but very few were able to identify successfully the values necessary to find the correct answer.

6a. *[2 marks]*

Markscheme

EITHER

$\theta = \pi - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$ (or equivalent) ***M1A1***

Note: Accept $\theta = 180^\circ - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$ (or equivalent).

OR

$\theta = \arctan\left(\frac{x}{8}\right) + \arctan\left(\frac{20-x}{13}\right)$ (or equivalent) ***M1A1***

[2 marks]

Examiners report

Part (a) was reasonably well done. While many candidates exhibited sound trigonometric knowledge to correctly express θ in terms of x, many other candidates were not able to use elementary trigonometry to formulate the required expression for θ.

6b. *[2 marks]*

Markscheme

(i) $\theta = 0.994 \ \left(= \arctan\frac{20}{13}\right)$ ***A1***

(ii) $\theta = 1.19 \ \left(= \arctan\frac{5}{2}\right)$ ***A1***

[2 marks]

Examiners report

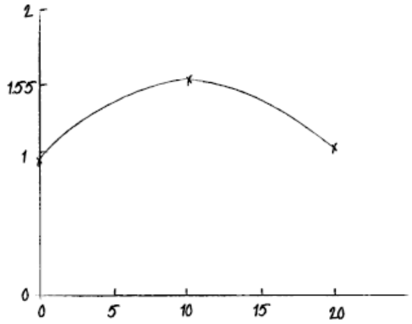
In part (b), a large number of candidates did not realize that θ could only be acute and gave obtuse angle values for θ. Many candidates also demonstrated a lack of insight when substituting endpoint x-values into θ.

6c. *[2 marks]*

Markscheme

correct shape. ***A1***

correct domain indicated. ***A1***



[2 marks]

Examiners report

In part (c), many candidates sketched either inaccurate or implausible graphs.

6d. *[6 marks]*

Markscheme

attempting to differentiate one $\arctan(f(x))$ term ***M1***

EITHER

$\theta = \pi - \arctan\left(\frac{8}{x}\right) - \arctan\left(\frac{13}{20-x}\right)$

$\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{8}{x^2} \times \frac{1}{1+\left(\frac{8}{x}\right)^2} - \frac{13}{(20-x)^2} \times \frac{1}{1+\left(\frac{13}{20-x}\right)^2}$ ***A1A1***

OR

$\theta = \arctan\left(\frac{x}{8}\right) + \arctan\left(\frac{20-x}{13}\right)$

$\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{\frac{1}{8}}{1+\left(\frac{x}{8}\right)^2} + \frac{-\frac{1}{13}}{1+\left(\frac{20-x}{13}\right)^2}$ ***A1A1***

THEN

$= \frac{8}{x^2+64} - \frac{13}{569-40x+x^2}$ ***A1***

$= \frac{8(569-40x+x^2)-13(x^2+64)}{(x^2+64)(x^2-40x+569)}$ ***M1A1***

$= \frac{5(744-64x-x^2)}{(x^2+64)(x^2-40x+569)}$ ***AG***

[6 marks]

Examiners report

In part (d), a large number of candidates started their differentiation incorrectly by failing to use the chain rule correctly.

6e. *[3 marks]*

Markscheme

Maximum light intensity at P occurs when $\frac{\mathrm{d}\theta}{\mathrm{d}x} = 0$. ***(M1)***

either attempting to solve $\frac{\mathrm{d}\theta}{\mathrm{d}x} = 0$ for *x* or using the graph of either θ or $\frac{\mathrm{d}\theta}{\mathrm{d}x}$ ***(M1)***

x = 10.05 (m) ***A1***

[3 marks]

Examiners report

For a question part situated at the end of the paper, part (e) was reasonably well done. A large number of candidates demonstrated a sound knowledge of finding where the maximum value of θ occurred and rejected solutions that were not physically feasible.

6f. *[4 marks]*

Markscheme

$\frac{\mathrm{d}x}{\mathrm{d}t} = 0.5$ ***(A1)***

At *x* = 10, $\frac{\mathrm{d}\theta}{\mathrm{d}x} = 0.000453 \ \left(= \frac{5}{11029}\right)$. ***(A1)***

use of $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\mathrm{d}\theta}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$ ***M1***

$\frac{\mathrm{d}\theta}{\mathrm{d}t} = 0.000227 \ \left(= \frac{5}{22058}\right) \ (\text{rad s}^{-1})$ ***A1***

Note: Award *(A1)* for $\frac{\mathrm{d}x}{\mathrm{d}t} = -0.5$ and ***A1*** for $\frac{\mathrm{d}\theta}{\mathrm{d}t} = -0.000227 \ \left(= -\frac{5}{22058}\right)$.

Note: Implicit differentiation can be used to find $\frac{\mathrm{d}\theta}{\mathrm{d}t}$. Award as above.

[4 marks]

Examiners report

In part (f), many candidates were able to link the required rates, however only a few candidates were able to successfully apply the chain rule in a related rates context.

7. *[6 marks]*

Markscheme

$V = \frac{\pi}{3} r^2 h$

$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\pi}{3}\left[2rh\frac{\mathrm{d}r}{\mathrm{d}t} + r^2\frac{\mathrm{d}h}{\mathrm{d}t}\right]$ ***M1A1A1***

at the given instant

$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\pi}{3}\left[2(4)(200)\left(-\frac{1}{2}\right) + 40^2(3)\right]$ ***M1***

$= \frac{-3200\pi}{3} = -3351.03\ldots \approx 3350$ ***A1***

hence, the volume is decreasing (at approximately 3350 mm^3 per century) ***R1***

[6 marks]

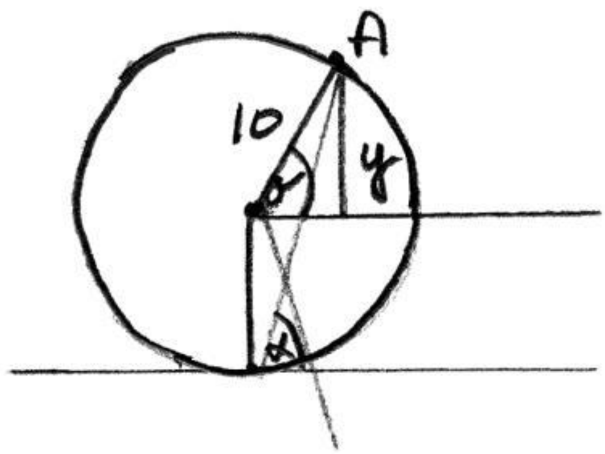
Examiners report

Few candidates applied the method of implicit differentiation and related rates correctly. Some candidates incorrectly interpreted this question as one of constant linear rates.

9. *[10 marks]*

Markscheme

(a)



$\frac{\mathrm{d}\theta}{\mathrm{d}t} = 3$ *(A1)*

$y = 10\sin\theta$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}\theta} = 10\cos\theta$ ***M1***

$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} = 30\cos\theta$ ***M1***

at $y = 6$, $\cos\theta = \frac{8}{10}$ ***(M1)(A1)***

$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} = 24$ (metres per minute) (accept 24.0) ***A1***

[7 marks]

(b) $\alpha = \frac{\theta}{2} + \frac{\pi}{4}$ ***M1A1***

$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \frac{1}{2}\frac{\mathrm{d}\theta}{\mathrm{d}t} = 1.5$ ***A1***

[3 marks]

Total [10 marks]

Examiners report

Many students were unable to get started with this question, and those that did were generally very poor at defining their variables at the start.

10. *[7 marks]*

Markscheme

(a) $\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{\mathrm{d}m}{\mathrm{d}y}\frac{\mathrm{d}y}{\mathrm{d}t}$ ***(M1)***

$= \sec^2\left(\arcsin\frac{y}{r}\right) \times \left(\arcsin\frac{y}{r}\right)' \times \frac{r}{1000}$

$= \frac{1}{\cos^2\left(\arcsin\frac{y}{r}\right)} \times \frac{\frac{1}{r}}{\sqrt{1-\left(\frac{y}{r}\right)^2}} \times \frac{r}{1000}$ (or equivalent) ***A1A1A1***

$= \frac{\frac{1}{\sqrt{r^2-y^2}}}{\frac{r^2-y^2}{r^2}}\frac{r}{1000}$ ***(A1)***

$= \frac{r^3}{10^3\sqrt{\left(r^2-y^2\right)^3}}$ (or equivalent) ***A1***

$= \left(\frac{r}{10\sqrt{r^2-y^2}}\right)^3$ ***AG N0***

(b) $\frac{\mathrm{d}m}{\mathrm{d}t}$ represents the rate of change of the gradient of the line OP ***A1***

[7 marks]

Examiners report

Few students were able to complete this question successfully, although many did obtain partial marks. Many students failed to recognise the difference between differentiating with respect to t or with respect to y. Very few were able to give a satisfactory geometrical meaning in part (b).

11. *[7 marks]*

Markscheme

(a) let $\mathrm{H\hat{P}Q} = \theta$

$\tan\theta = \frac{h}{40}$

$\sec^2\theta\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{1}{40}\frac{\mathrm{d}h}{\mathrm{d}t}$ ***M1***

$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{1}{4\sec^2\theta}$ ***(A1)***

$= \frac{16}{4\times25}\left(\sec\theta = \frac{5}{4}\ \text{ or } \theta = 0.6435\right)$ ***A1***

$= 0.16$ radians per second ***AG***

(b) $x^2 = h^2 + 1600$, where $\mathrm{PH} = x$

$2x\frac{\mathrm{d}x}{\mathrm{d}t} = 2h\frac{\mathrm{d}h}{\mathrm{d}t}$ ***M1***

$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{h}{x} \times 10$ ***A1***

$= \frac{10h}{\sqrt{h^2+1600}}$ ***(A1)***

$h = 30$, $\frac{\mathrm{d}x}{\mathrm{d}t} = 6$ ms***A1***

Note: Accept solutions that begin $x = 40\sec\theta$ or use $h = 10t$.

[7 marks]

Examiners report

For those candidates who realized this was an applied calculus problem involving related rates of change, the main source of error was in differentiating inverse tan in part (a). Some found part (b) easier than part (a), involving a changing length rather than an angle. A number of alternative approaches were reported by examiners.

13. *[5 marks]*

Markscheme

METHOD 1

volume of a cone is $V = \frac{1}{3}\pi r^2 h$

given $h = r,\ V = \frac{1}{3}\pi h^3$ ***M1***

$\frac{\mathrm{d}V}{\mathrm{d}h} = \pi h^2$ ***(A1)***

when $h = 4,\ \frac{\mathrm{d}V}{\mathrm{d}t} = \pi \times 4^2 \times 0.5$ (using $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t}$) ***M1A1***

$\frac{\mathrm{d}V}{\mathrm{d}t} = 8\pi\ (= 25.1)\ (\mathrm{cm}^3\ \mathrm{min}^{-1})$ ***A1***

METHOD 2

volume of a cone is $V = \frac{1}{3}\pi r^2 h$

given $h = r,\ V = \frac{1}{3}\pi h^3$ ***M1***

$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1}{3}\pi \times 3h^2 \times \frac{\mathrm{d}h}{\mathrm{d}t}$ ***A1***

when $h = 4,\ \frac{\mathrm{d}V}{\mathrm{d}t} = \pi \times 4^2 \times 0.5$ ***M1A1***

$\frac{\mathrm{d}V}{\mathrm{d}t} = 8\pi\ (= 25.1)\ (\mathrm{cm}^3\ \mathrm{min}^{-1})$ ***A1***

METHOD 3

$V = \frac{1}{3}\pi r^2 h$

$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1}{3}\pi\left(2rh\frac{\mathrm{d}r}{\mathrm{d}t} + r^2\frac{\mathrm{d}h}{\mathrm{d}t}\right)$ ***M1A1***

Note: Award ***M1*** for attempted implicit differentiation and ***A1*** for each correct term on the RHS.

when $h = 4,\ r = 4,\ \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1}{3}\pi\left(2 \times 4 \times 4 \times 0.5 + 4^2 \times 0.5\right)$ ***M1A1***

$\frac{\mathrm{d}V}{\mathrm{d}t} = 8\pi\ (= 25.1)\ (\mathrm{cm}^3\ \mathrm{min}^{-1})$ ***A1***

[5 marks]