Sequences 2008-2014 with MS

1. *[4 marks]*

Find the value of *k* if $\sum_{r=1}^{\infty} k\left(\frac{1}{3}\right)^r = 7$.

2a. *[4 marks]*

The sum of the first 16 terms of an arithmetic sequence is 212 and the fifth term is 8.

Find the first term and the common difference.

2b. *[3 marks]*

Find the smallest value of *n* such that the sum of the first *n* terms is greater than 600.

3a. *[2 marks]*

Each time a ball bounces, it reaches 95 % of the height reached on the previous bounce.

Initially, it is dropped from a height of 4 metres.

What height does the ball reach after its fourth bounce?

3b. *[3 marks]*

How many times does the ball bounce before it no longer reaches a height of 1 metre?

3c. *[3 marks]*

What is the total distance travelled by the ball?

4. *[4 marks]*

Find the sum of all the multiples of 3 between 100 and 500.

5. *[6 marks]*

A metal rod 1 metre long is cut into 10 pieces, the lengths of which form a geometric sequence. The length of the longest piece is 8 times the length of the shortest piece. Find, to the nearest millimetre, the length of the shortest piece.

6a. *[3 marks]*

An arithmetic sequence has first term *a* and common difference *d*, $d \neq 0$. The 3^{rd}, 4^{th} and 7^{th} terms of the arithmetic sequence are the first three terms of a geometric sequence.

Show that $a = -\frac{3}{2}d$.

6b. *[5 marks]*

Show that the 4^{th} term of the geometric sequence is the 16^{th} term of the arithmetic sequence.

7. *[5 marks]*

In the arithmetic series with n^{th} term u_n , it is given that $u_4 = 7$ and $u_9 = 22$.

Find the minimum value of *n* so that $u_1 + u_2 + u_3 + \ldots + u_n > 10\,000$.

8a. *[6 marks]*

The integral I_n is defined by $I_n = \int_{n\pi}^{(n+1)\pi} \mathrm{e}^{-x} |\sin x| \mathrm{d}x, \text{ for } n \in \mathbb{N}$.

Show that $I_0 = \frac{1}{2}\left(1 + \mathrm{e}^{-\pi}\right)$.

8b. *[4 marks]*

By letting $y = x - n\pi$, show that $I_n = \mathrm{e}^{-n\pi} I_0$.

8c. *[5 marks]*

Hence determine the exact value of $\int_0^{\infty} \mathrm{e}^{-x} |\sin x| \mathrm{d}x$.

9. *[6 marks]*

The first terms of an arithmetic sequence are $\frac{1}{\log_2 x}, \frac{1}{\log_8 x}, \frac{1}{\log_{32} x}, \frac{1}{\log_{128} x}, \ldots$

Find *x* if the sum of the first 20 terms of the sequence is equal to 100.

10. *[6 marks]*

The mean of the first ten terms of an arithmetic sequence is 6. The mean of the first twenty terms of the arithmetic sequence is 16. Find the value of the 15^{th} term of the sequence.

11. *[7 marks]*

A geometric sequence has first term *a*, common ratio *r* and sum to infinity 76. A second geometric sequence has first term *a*, common ratio r^3 and sum to infinity 36.

Find *r*.

12a. *[2 marks]*

The arithmetic sequence $\{u_n : n \in \mathbb{Z}^+\}$ has first term $u_1 = 1.6$ and common difference *d* = 1.5. The geometric sequence $\{v_n : n \in \mathbb{Z}^+\}$ has first term $v_1 = 3$ and common ratio *r* = 1.2.

Find an expression for $u_n - v_n$ in terms of *n*.

12b. *[3 marks]*

Determine the set of values of *n* for which $u_n > v_n$.

12c. *[1 mark]*

Determine the greatest value of $u_n - v_n$. Give your answer correct to four significant figures.

13a. *[4 marks]*

(i) Express the sum of the first *n* positive odd integers using sigma notation.

(ii) Show that the sum stated above is n^2.

(iii) Deduce the value of the difference between the sum of the first 47 positive odd integers and the sum of the first 14 positive odd integers.

13b. *[7 marks]*

A number of distinct points are marked on the circumference of a circle, forming a polygon. Diagonals are drawn by joining all pairs of non-adjacent points.

(i) Show on a diagram all diagonals if there are 5 points.

(ii) Show that the number of diagonals is $\frac{n(n-3)}{2}$ if there are n points, where $n > 2$.

(iii) Given that there are more than one million diagonals, determine the least number of points for which this is possible.

13c. *[8 marks]*

The random variable $X \sim B(n,\ p)$ has mean 4 and variance 3.

(i) Determine *n* and *p*.

(ii) Find the probability that in a single experiment the outcome is 1 or 3.

14a. *[4 marks]*

Find the set of values of *x* for which the series $\sum\limits_{n=1}^{\infty} \left(\frac{2x}{x+1}\right)^n$ has a finite sum.

14b. *[2 marks]*

Hence find the sum in terms of *x*.

15a. *[9 marks]*

In an arithmetic sequence the first term is 8 and the common difference is $\frac{1}{4}$. If the sum of the first 2*n* terms is equal to the sum of the next *n* terms, find *n*.

15b. *[7 marks]*

If $a_1,\ a_2,\ a_3,\ \ldots$ are terms of a geometric sequence with common ratio $r \neq 1$, show that $(a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_4)^2 + \ldots + (a_n - a_{n+1})^2 = \frac{a_1^2(1-r)(1-r^{2n})}{1+r}$.

16a. *[1 mark]*

Show that $\left|\mathrm{e}^{\mathrm{i}\theta}\right| = 1$.

16b. *[2 marks]*

Consider the geometric series $1 + \frac{1}{3}\mathrm{e}^{\mathrm{i}\theta} + \frac{1}{9}\mathrm{e}^{2\mathrm{i}\theta} + \ldots$.

Write down the common ratio, *z*, of the series, and show that $|z| = \frac{1}{3}$.

16c. *[2 marks]*

Find an expression for the sum to infinity of this series.

16d. *[8 marks]*

Hence, show that $\sin\theta + \frac{1}{3}\sin 2\theta + \frac{1}{9}\sin 3\theta + \ldots = \frac{9\sin\theta}{10-6\cos\theta}$.

17a. *[2 marks]*

A geometric sequence $u_1, u_2, u_3, \ldots$ has $u_1 = 27$ and a sum to infinity of $\frac{81}{2}$.

Find the common ratio of the geometric sequence.

17b. *[5 marks]*

An arithmetic sequence $v_1, v_2, v_3, \ldots$ is such that $v_2 = u_2$ and $v_4 = u_4$.

Find the greatest value of N such that $\sum_{n=1}^{N} v_n > 0$.

18. *[17 marks]*

A geometric sequence $\{u_n\}$, with complex terms, is defined by $u_{n+1} = (1+\mathrm{i})u_n$ and $u_1 = 3$.

(a) Find the fourth term of the sequence, giving your answer in the form $x + y\mathrm{i},\ x,\ y \in \mathbb{R}$.

(b) Find the sum of the first 20 terms of $\{u_n\}$, giving your answer in the form $a \times (1 + 2^m)$ where $a \in \mathbb{C}$ and $m \in \mathbb{Z}$ are to be determined.

A second sequence $\{v_n\}$ is defined by $v_n = u_n u_{n+k},\ k \in \mathbb{N}$.

(c) (i) Show that $\{v_n\}$ is a geometric sequence.

(ii) State the first term.

(iii) Show that the common ratio is independent of *k*.

A third sequence $\{w_n\}$ is defined by $w_n = |u_n - u_{n+1}|$.

(d) (i) Show that $\{w_n\}$ is a geometric sequence.

(ii) State the geometrical significance of this result with reference to points on the complex plane.

19. *[7 marks]*

The first three terms of a geometric sequence are $\sin x,\ \sin 2x$ and $4\sin x\cos^2 x,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$.

(a) Find the common ratio *r*.

(b) Find the set of values of *x* for which the geometric series $\sin x + \sin 2x + 4\sin x\cos^2 x + \ldots$ converges.

Consider $x = \arccos\left(\frac{1}{4}\right),\ x > 0$.

(c) Show that the sum to infinity of this series is $\frac{\sqrt{15}}{2}$.

20. *[6 marks]*

(a) (i) Find the sum of all integers, between 10 and 200, which are divisible by 7.

(ii) Express the above sum using sigma notation.

An arithmetic sequence has first term 1000 and common difference of −6 . The sum of the first *n* terms of this sequence is negative.

(b) Find the least value of *n*.

21. *[7 marks]*

The sum of the first two terms of a geometric series is 10 and the sum of the first four terms is 30.

(a) Show that the common ratio r satisfies $r^2 = 2$.

(b) Given $r = \sqrt{2}$

(i) find the first term;

(ii) find the sum of the first ten terms.

22. *[7 marks]*

The fourth term in an arithmetic sequence is 34 and the tenth term is 76.

(a) Find the first term and the common difference.

(b) The sum of the first *n* terms exceeds 5000. Find the least possible value of *n*.

Sequences 2008-2014 MS

1. *[4 marks]*

Markscheme

$u_1 = \frac{1}{3}k, \; r = \frac{1}{3}$ ***(A1) (A1)***

$7 = \frac{\frac{1}{3}k}{1-\frac{1}{3}}$ ***M1***

$k = 14$ ***A1***

[4 marks]

Examiners report

The question was well done generally. Those that did make mistakes on the question usually had the first term wrong, but did understand to use the formula for an infinite geometric series.

2a. *[4 marks]*

Markscheme

$S_n = \frac{n}{2}[2a + (n-1)d]$

$212 = \frac{16}{2}(2a + 15d) \quad (= 16a + 120d)$ ***A1***

n^{th} term is $a + (n-1)d$

$8 = a + 4d$ ***A1***

solving simultaneously: ***(M1)***

$d = 1.5, \; a = 2$ ***A1***

[4 marks]

Examiners report

This proved to be a good start to the paper for most candidates. The vast majority made a meaningful attempt at this question with many gaining the correct answers. Candidates who lost marks usually did so because of mistakes in the working. In part (b) the most efficient way of gaining the answer was to use the calculator once the initial inequality was set up. A small number of candidates spent valuable time unnecessarily manipulating the algebra before moving to the calculator.

2b. *[3 marks]*

Markscheme

$\frac{n}{2}[4 + 1.5(n-1)] > 600$ ***(M1)***

$\Rightarrow 3n^2 + 5n - 2400 > 0$ ***(A1)***

$\Rightarrow n > 27.4\ldots,\ (n < -29.1\ldots)$

Note: Do not penalize improper use of inequalities.

$\Rightarrow n = 28$ ***A1***

[3 marks]

Examiners report

This proved to be a good start to the paper for most candidates. The vast majority made a meaningful attempt at this question with many gaining the correct answers. Candidates who lost marks usually did so because of mistakes in the working. In part (b) the most efficient way of gaining the answer was to use the calculator once the initial inequality was set up. A small number of candidates spent valuable time unnecessarily manipulating the algebra before moving to the calculator.

3a. *[2 marks]*

Markscheme

height $= 4 \times 0.95^4$ ***(A1)***

= 3.26 (metres) ***A1***

[2 marks]

Examiners report

The majority of candidates were able to start this question and gain some marks, but only better candidates gained full marks. In part (a) the common error was to assume the wrong number of bounces and in part (b) many candidates lost marks due to rounding the inequality in the wrong direction. Part (c) was found difficult with only a limited number recognising the need for the sum to infinity of a geometric sequence and many of those did not recognise how to link the sum to infinity to the total distance travelled.

3b. *[3 marks]*

Markscheme

$4 \times 0.95^n < 1$ ***(M1)***

$0.95^n < 0.25$

$\Rightarrow n > \frac{\ln 0.25}{\ln 0.95}$ ***(A1)***

$\Rightarrow n > 27.0$

Note: Do not penalize improper use of inequalities.

$\Rightarrow n = 28$ ***A1***

Note: If candidates have used *n* – 1 rather than *n* throughout penalise in part (a) and treat as follow through in parts (b) and (c).

[3 marks]

Examiners report

The majority of candidates were able to start this question and gain some marks, but only better candidates gained full marks. In part (a) the common error was to assume the wrong number of bounces and in part (b) many candidates lost marks due to rounding the inequality in the wrong direction. Part (c) was found difficult with only a limited number recognising the need for the sum to infinity of a geometric sequence and many of those did not recognise how to link the sum to infinity to the total distance travelled.

3c. *[3 marks]*

Markscheme

METHOD 1

recognition of geometric series with sum to infinity, first term of 4×0.95 and common ratio 0.95 ***M1***

recognition of the need to double this series and to add 4 ***M1***

total distance travelled is $2\left(\frac{4\times0.95}{1-0.95}\right)+4=156\text{ (metres)}$ ***A1***

[3 marks]

Note: If candidates have used *n* – 1 rather than *n* throughout penalise in part (a) and treat as follow through in parts (b) and (c).

METHOD 2

recognition of a geometric series with sum to infinity, first term of 4 and common ratio 0.95 **M1**

recognition of the need to double this series and to subtract 4 **M1**

total distance travelled is $2\left(\frac{4}{1-0.95}\right)-4=156\text{ (metres)}$ **A1**

[3 marks]

Examiners report

The majority of candidates were able to start this question and gain some marks, but only better candidates gained full marks. In part (a) the common error was to assume the wrong number of bounces and in part (b) many candidates lost marks due to rounding the inequality in the wrong direction. Part (c) was found difficult with only a limited number recognising the need for the sum to infinity of a geometric sequence and many of those did not recognise how to link the sum to infinity to the total distance travelled.

4. *[4 marks]*

Markscheme

METHOD 1

102 + 105 + ... + 498 ***(M1)***

so number of terms = 133 ***(A1)***

EITHER

$= \frac{133}{2}\left(2 \times 102 + 132 \times 3\right)$ ***(M1)***

= 39900 ***A1***

OR

$= \left(102 + 498\right) \times \frac{133}{2}$ ***(M1)***

= 39900 ***A1***

OR

$\sum\limits_{n=34}^{166} 3n$ ***(M1)***

= 39900 ***A1***

METHOD 2

$500 \div 3 = 166.666\ldots \text{ and } 100 \div 3 = 33.333\ldots$

$102 + 105 + \ldots + 498 = \sum\limits_{n=1}^{166} 3n - \sum\limits_{n=1}^{33} 3n$ ***(M1)***

$\sum\limits_{n=1}^{166} 3n = 41583$ ***(A1)***

$\sum\limits_{n=1}^{33} 3n = 1683$ ***(A1)***

the sum is 39900 ***A1***

[4 marks]

Examiners report

Most candidates got full marks in this question. Some mistakes were detected when trying to find the number of terms of the arithmetic sequence, namely the use of the incorrect value $n = 132$; a few interpreted the question as the sum of multiples between the 100th and 500th terms. Occasional application of geometric series was attempted.

5. *[6 marks]*

Markscheme

the pieces have lengths $a,\ ar,\ \ldots,\ ar^9$ ***(M1)***

$8a = ar^9 \ (\text{or } 8 = r^9)$ ***A1***

$r = \sqrt[9]{8} = 1.259922\ldots$ ***A1***

$a\,\frac{r^{10}-1}{r-1} = 1 \quad \left(\text{or } a\,\frac{r^{10}-1}{r-1} = 1000\right)$ ***M1***

$a = \frac{r-1}{r^{10}-1} = 0.0286\ldots \quad \left(\text{or } a = \frac{r-1}{r^{10}-1} = 28.6\ldots\right)$ ***(A1)***

a = 29 mm (accept 0.029 m or any correct answer regardless the units) ***A1***

[6 marks]

Examiners report

This question was generally well done by most candidates. Some candidates recurred to a diagram to comprehend the nature of the problem but a few thought it was an arithmetic sequence.

A surprising number of candidates missed earning the final A1 mark because they did not read the question instructions fully and missed the accuracy instruction to give the answer correct to the nearest mm.

6a. *[3 marks]*

Markscheme

let the first three terms of the geometric sequence be given by u_1 , $u_1 r$, $u_1 r^2$

$\therefore u_1 = a + 2d$, $u_1 r = a + 3d$ and $u_1 r^2 = a + 6d$ ***(M1)***

$\frac{a+6d}{a+3d} = \frac{a+3d}{a+2d}$ ***A1***

$a^2 + 8ad + 12d^2 = a^2 + 6ad + 9d^2$ ***A1***

2*a* + 3*d* = 0

$a = -\frac{3}{2}d$ ***AG***

[3 marks]

Examiners report

This question was done well by many students. Those who did not do it well often became involved in convoluted algebraic processes that complicated matters significantly. There were a number of different approaches taken which were valid.

6b. *[5 marks]*

Markscheme

$u_1 = \frac{d}{2}$, $u_1 r = \frac{3d}{2}$, $\left(u_1 r^2 = \frac{9d}{2}\right)$ ***M1***

r = 3 ***A1***

geometric 4^{th} term $u_1 r^3 = \frac{27d}{2}$ ***A1***

arithmetic 16^{th} term $a + 15d = -\frac{3}{2}d + 15d$ ***M1***

$= \frac{27d}{2}$ ***A1***

Note: Accept alternative methods.

[3 marks]

Examiners report

This question was done well by many students. Those who did not do it well often became involved in convoluted algebraic processes that complicated matters significantly. There were a number of different approaches taken which were valid.

7. *[5 marks]*

Markscheme

$u_4 = u_1 + 3d = 7$, $u_9 = u_1 + 8d = 22$ ***A1A1***

Note: $5d = 15$ gains both above marks

$u_1 = -2$, $d = 3$ ***A1***

$S_n = \frac{n}{2}(-4 + (n-1)3) > 10\,000$ ***M1***

$n = 83$ ***A1***

[5 marks]

Examiners report

This question was well answered by most candidates. A few did not realise that the answer had to be an integer.

8a. *[6 marks]*

Markscheme

$I_0 = \int_0^\pi \mathrm{e}^{-x} \sin x \mathrm{d}x$ ***M1***

Note: Award ***M1*** for $I_0 = \int_0^\pi \mathrm{e}^{-x} |\sin x| \mathrm{d}x$

Attempt at integration by parts, even if inappropriate modulus signs are present. ***M1***

$= -[\mathrm{e}^{-x} \cos x]_0^\pi - \int_0^\pi \mathrm{e}^{-x} \cos x \mathrm{d}x$ **or** $= -[\mathrm{e}^{-x} \sin x]_0^\pi - \int_0^\pi \mathrm{e}^{-x} \cos x \mathrm{d}x$ ***A1***

$= -[\mathrm{e}^{-x} \cos x]_0^\pi - [\mathrm{e}^{-x} \sin x]_0^\pi - \int_0^\pi \mathrm{e}^{-x} \sin x \mathrm{d}x$ **or**
$= -[\mathrm{e}^{-x} \sin x + \mathrm{e}^{-x} \cos x]_0^\pi - \int_0^\pi \mathrm{e}^{-x} \sin x \mathrm{d}x$ ***A1***

$= -[\mathrm{e}^{-x} \cos x]_0^\pi - [\mathrm{e}^{-x} \sin x]_0^\pi - I_0$ **or** $-[\mathrm{e}^{-x} \sin x + \mathrm{e}^{-x} \cos x]_0^\pi - I_0$ ***M1***

Note: Do not penalise absence of limits at this stage

$I_0 = \mathrm{e}^{-\pi} + 1 - I_0$ ***A1***

$I_0 = \frac{1}{2}\left(1 + \mathrm{e}^{-\pi}\right)$ ***AG***

Note: If modulus signs are used around cos *x* , award no accuracy marks but do not penalise modulus signs around sin *x* .

[6 marks]

Examiners report

Part (a) is essentially core work requiring repeated integration by parts and many candidates realised that. However, some candidates left the modulus signs in I_0 which invalidated their work. In parts (b) and (c) it was clear that very few candidates had a complete understanding of the significance of the modulus sign and what conditions were necessary for it to be dropped. Overall, attempts at (b) and (c) were disappointing with few correct solutions seen.

8b. *[4 marks]*

Markscheme

$I_n = \int_{n\pi}^{(n+1)\pi} \mathrm{e}^{-x}|\sin x|\mathrm{d}x$

Attempt to use the substitution $y = x - n\pi$ ***M1***

(putting $y = x - n\pi$, $\mathrm{d}y = \mathrm{d}x$ and $[n\pi,\ (n+1)\pi] \to [0,\ \pi]$)

so $I_n = \int_0^{\pi} \mathrm{e}^{-(y+n\pi)}|\sin(y + n\pi)|\mathrm{d}y$ ***A1***

$= \mathrm{e}^{-n\pi}\int_0^{\pi} \mathrm{e}^{-y}|\sin(y + n\pi)|\mathrm{d}y$ ***A1***

$= \mathrm{e}^{-n\pi}\int_0^{\pi} \mathrm{e}^{-y}\sin y\mathrm{d}y$ ***A1***

$= \mathrm{e}^{-n\pi}I_0$ ***AG***

[4 marks]

Examiners report

Part (a) is essentially core work requiring repeated integration by parts and many candidates realised that. However, some candidates left the modulus signs in I_0 which invalidated their work. In parts (b) and (c) it was clear that very few candidates had a complete understanding of the significance of the modulus sign and what conditions were necessary for it to be dropped. Overall, attempts at (b) and (c) were disappointing with few correct solutions seen.

8c. *[5 marks]*

Markscheme

$\int_0^{\infty} \mathrm{e}^{-x}|\sin x|\mathrm{d}x = \sum_{n=0}^{\infty} I_n$ ***M1***

$= \sum_{n=0}^{\infty} \mathrm{e}^{-n\pi}I_0$ ***(A1)*** the $\sum$ term is an infinite geometric series with common ratio $\mathrm{e}^{-\pi}$ ***(M1)***

therefore

$\int_0^\infty \mathrm{e}^{-x}|\sin x|\mathrm{d}x = \frac{I_0}{1-\mathrm{e}^{-\pi}}$ ***(A1)***

$= \frac{1+\mathrm{e}^{-\pi}}{2(1-\mathrm{e}^{-\pi})} \quad \left(= \frac{\mathrm{e}^{\pi}+1}{2(\mathrm{e}^{\pi}-1)}\right)$ ***A1***

[5 marks]

Examiners report

Part (a) is essentially core work requiring repeated integration by parts and many candidates realised that. However, some candidates left the modulus signs in I_0 which invalidated their work. In parts (b) and (c) it was clear that very few candidates had a complete understanding of the significance of the modulus sign and what conditions were necessary for it to be dropped. Overall, attempts at (b) and (c) were disappointing with few correct solutions seen.

9. *[6 marks]*

Markscheme

METHOD 1

$d = \frac{1}{\log_8 x} - \frac{1}{\log_2 x}$ ***(M1)***

$= \frac{\log_2 8}{\log_2 x} - \frac{1}{\log_2 x}$ ***(M1)***

Note: Award this ***M1*** for a correct change of base anywhere in the question.

$= \frac{2}{\log_2 x}$ ***(A1)***

$\frac{20}{2}\left(2 \times \frac{1}{\log_2 x} + 19 \times \frac{2}{\log_2 x}\right)$ ***M1***

$= \frac{400}{\log_2 x}$ ***(A1)***

$100 = \frac{400}{\log_2 x}$

$\log_2 x = 4 \Rightarrow x = 2^4 = 16$ ***A1***

METHOD 2

$20^{\text{th}}\text{ term} = \frac{1}{\log_{2^{39}} x}$ ***A1***

$100 = \frac{20}{2}\left(\frac{1}{\log_2 x} + \frac{1}{\log_{2^{39}} x}\right)$ ***M1***

$100 = \frac{20}{2}\left(\frac{1}{\log_2 x} + \frac{\log_2 2^{39}}{\log_2 x}\right)$ ***M1(A1)***

Note: Award this ***M1*** for a correct change of base anywhere in the question.

$100 = \frac{400}{\log_2 x}$ ***(A1)***

$\log_2 x = 4 \Rightarrow x = 2^4 = 16$ ***A1***

METHOD 3

$\frac{1}{\log_2 x} + \frac{1}{\log_8 x} + \frac{1}{\log_{32} x} + \frac{1}{\log_{128} x} + \ldots$

$\frac{1}{\log_2 x} + \frac{\log_2 8}{\log_2 x} + \frac{\log_2 32}{\log_2 x} + \frac{\log_2 128}{\log_2 x} + \ldots$ ***(M1)(A1)***

Note: Award this ***M1*** for a correct change of base anywhere in the question.

$= \frac{1}{\log_2 x}(1 + 3 + 5 + \ldots)$ ***A1***

$= \frac{1}{\log_2 x}\left(\frac{20}{2}(2 + 38)\right)$ ***(M1)(A1)***

$100 = \frac{400}{\log_2 x}$

$\log_2 x = 4 \Rightarrow x = 2^4 = 16$ ***A1***

[6 marks]

Examiners report

There were plenty of good answers to this question. Those who realised they needed to make each log have the same base (and a great variety of bases were chosen) managed the question successfully.

10. *[6 marks]*

Markscheme

METHOD 1

$5(2a + 9d) = 60$ (or $2a + 9d = 12$) ***M1A1***

$10(2a + 19d) = 320$ (or $2a + 19d = 32$) ***A1***

solve simultaneously to obtain ***M1***

$a = -3,\ d = 2$ ***A1***

the 15^{th} term is $-3 + 14 \times 2 = 25$ ***A1***

Note: ***FT*** the final ***A1*** on the values found in the penultimate line.

METHOD 2

with an AP the mean of an even number of consecutive terms equals the mean of the middle terms ***(M1)***

$\frac{a_{10}+a_{11}}{2} = 16 \quad (\text{or } a_{10} + a_{11} = 32)$ ***A1***

$\frac{a_5+a_6}{2} = 6 \quad (\text{or } a_5 + a_6 = 12)$ ***A1***

$a_{10} - a_5 + a_{11} - a_6 = 20$ ***M1***

$5d + 5d = 20$

$d = 2 \text{ and } a = -3 \quad (\text{or } a_5 = 5 \text{ or } a_{10} = 15)$ ***A1***

the 15^{th} term is $-3 + 14 \times 2 = 25 \quad (\text{or } 5 + 10 \times 2 = 25 \text{ or } 15 + 5 \times 2 = 25)$ ***A1***

Note: ***FT*** the final ***A1*** on the values found in the penultimate line.

[6 marks]

Examiners report

Many candidates had difficulties with this question with the given information often translated into incorrect equations.

11. *[7 marks]*

Markscheme

for the first series $\frac{a}{1-r} = 76$ ***A1***

for the second series $\frac{a}{1-r^3} = 36$ ***A1***

attempt to eliminate *a e.g.* $\frac{76(1-r)}{1-r^3} = 36$ ***M1***

simplify and obtain $9r^2 + 9r - 10 = 0$ ***(M1)A1***

Note: Only award the ***M1*** if a quadratic is seen.

obtain $r = \frac{12}{18}$ and $-\frac{30}{18}$ ***(A1)***

$r = \frac{12}{18} \left(= \frac{2}{3} = 0.666\ldots\right)$ ***A1***

Note: Award ***A0*** if the extra value of *r* is given in the final answer.

Total [7 marks]

Examiners report

Almost all candidates obtained the cubic equation satisfied by the common ratio of the first sequence, but few were able to find its roots. One of the roots was $r = 1$.

12a. *[2 marks]*

Markscheme

$u_n - v_n = 1.6 + (n-1) \times 1.5 - 3 \times 1.2^{n-1}$ $(= 1.5n + 0.1 - 3 \times 1.2^{n-1})$ ***A1A1***

[2 marks]

Examiners report

In part (a), most candidates were able to express u_n and v_n correctly and hence obtain a correct expression for $u_n - v_n$. Some candidates made careless algebraic errors when unnecessarily simplifying u_n while other candidates incorrectly stated v_n as $3(1.2)^n$.

12b. *[3 marks]*

Markscheme

attempting to solve $u_n > v_n$ numerically or graphically. ***(M1)***

$n = 2.621\ldots, 9.695\ldots$ ***(A1)***

So $3 \leqslant n \leqslant 9$ ***A1***

[3 marks]

Examiners report

In parts (b) and (c), most candidates treated *n* as a continuous variable rather than as a discrete variable. Candidates should be aware that a GDC's table feature can be extremely useful when attempting such question types.

12c. *[1 mark]*

Markscheme

The greatest value of $u_n - v_n$ is 1.642. ***A1***

Note: Do not accept 1.64.

[1 mark]

Examiners report

In parts (b) and (c), most candidates treated *n* as a continuous variable rather than as a discrete variable. Candidates should be aware that a GDC's table feature can be extremely useful when attempting such question types. In part (c), a number of candidates attempted to find the maximum value of *n* rather than attempting to find the maximum value of $u_n - v_n$.

13a. *[4 marks]*

Markscheme

(i) $\sum\limits_{k=1}^{n} (2k-1)$ (or equivalent) ***A1***

Note: Award ***A0*** for $\sum\limits_{n=1}^{n} (2n-1)$ or equivalent.

(ii) **EITHER**

$2 \times \frac{n(n+1)}{2} - n$ ***M1A1***

OR

$\frac{n}{2}(2 + (n-1)2)$ (using $S_n = \frac{n}{2}(2u_1 + (n-1)d)$) ***M1A1***

OR

$\frac{n}{2}(1 + 2n - 1)$ (using $S_n = \frac{n}{2}(u_1 + u_n)$) ***M1A1***

THEN

$= n^2$ ***AG***

(iii) $47^2 - 14^2 = 2013$ ***A1***

[4 marks]

Examiners report

In part (a) (i), a large number of candidates were unable to correctly use sigma notation to express the sum of the first *n* positive odd integers. Common errors included summing $2n - 1$ from 1 to *n* and specifying sums with incorrect limits. Parts (a) (ii) and (iii) were generally well done.

13b. *[7 marks]*

Markscheme

(i) **EITHER**

a pentagon and five diagonals ***A1***

OR

five diagonals (circle optional) ***A1***

(ii) Each point joins to *n* – 3 other points. ***A1***

a correct argument for $n(n-3)$ ***R1***

a correct argument for $\frac{n(n-3)}{2}$ ***R1***

(iii) attempting to solve $\frac{1}{2}n(n-3) > 1\,000\,000$ for *n*. ***(M1)***

$n > 1415.7$ ***(A1)***

$n = 1416$ ***A1***

[7 marks]

Examiners report

Parts (b) (i) and (iii) were generally well done. In part (b) (iii), many candidates unnecessarily simplified their quadratic when direct GDC use could have been employed. A few candidates gave $n > 1416$ as their final answer. While some candidates displayed sound reasoning in part (b) (ii), many candidates unfortunately adopted a 'proof by example' approach.

13c. *[8 marks]*

Markscheme

(i) *np* = 4 and *npq* = 3 ***(A1)***

attempting to solve for *n* and *p* ***(M1)***

$n = 16$ and $p = \frac{1}{4}$ ***A1***

(ii) $X \sim B(16, 0.25)$ ***(A1)***

$P(X = 1) = 0.0534538...(= \binom{16}{1}(0.25)(0.75)^{15})$ ***(A1)***

$P(X = 3) = 0.207876...(= \binom{16}{3}(0.25)^3(0.75)^{13})$ ***(A1)***

$\mathrm{P}(X = 1) + \mathrm{P}(X = 3)$ ***(M1)***

= 0.261 ***A1***

[8 marks]

Examiners report

Part (c) was generally well done. In part (c) (ii), some candidates multiplied the two probabilities rather than adding the two probabilities.

14a. *[4 marks]*

Markscheme

for the series to have a finite sum, $\left|\frac{2x}{x+1}\right| < 1$ ***R1***

(sketch from gdc or algebraic method) ***M1***

S_∞ exists when $-\frac{1}{3} < x < 1$ ***A1A1***

Note: Award ***A1*** for bounds and ***A1*** for strict inequalities.

[4 marks]

Examiners report

A large number of candidates omitted the absolute value sign in the inequality in (a), or the use of the correct double inequality. Among candidates who had the correct statement, those who used their GDC were the most successful. The algebraic solution of the inequality was difficult for some candidates. In (b), quite a number of candidates found the sum of the first *n* terms of the geometric series, rather than the infinite sum of the series.

14b. *[2 marks]*

Markscheme

$S_\infty = \frac{\frac{2x}{x+1}}{1-\frac{2x}{x+1}} = \frac{2x}{1-x}$ ***M1A1***

[2 marks]

Examiners report

A large number of candidates omitted the absolute value sign in the inequality in (a), or the use of the correct double inequality. Among candidates who had the correct statement, those who used their GDC were the most successful. The algebraic solution of the inequality was difficult for some candidates. In (b), quite a number of candidates found the sum of the first *n* terms of the geometric series, rather than the infinite sum of the series.

15a. *[9 marks]*

Markscheme

$S_{2n} = \frac{2n}{2}\left(2(8) + (2n-1)\frac{1}{4}\right)$ ***(M1)***

$= n\left(16 + \frac{2n-1}{4}\right)$ ***A1***

$S_{3n} = \frac{3n}{2}\left(2 \times 8 + (3n-1)\frac{1}{4}\right)$ ***(M1)***

$= \frac{3n}{2}\left(16 + \frac{3n-1}{4}\right)$ ***A1***

$S_{2n} = S_{3n} - S_{2n} \Rightarrow 2S_{2n} = S_{3n}$ ***M1***

solve $2S_{2n} = S_{3n}$

$\Rightarrow 2n\left(16 + \frac{2n-1}{4}\right) = \frac{3n}{2}\left(16 + \frac{3n-1}{4}\right)$ ***A1***

$\left(\Rightarrow 2\left(16 + \frac{2n-1}{4}\right) = \frac{3}{2}\left(16 + \frac{3n-1}{4}\right)\right)$

(gdc or algebraic solution) ***(M1)***

n = 63 ***A2***

[9 marks]

Examiners report

Many candidates were able to solve (a) successfully. A few candidates failed to understand the relationship between S_{2n} and S_{3n}, and hence did not obtain the correct equation. (b) was answered poorly by a large number of candidates. There was significant difficulty in forming correct general statements, and a general lack of rigor in providing justification.

15b. *[7 marks]*

Markscheme

$(a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_4)^2 + \ldots$

$= (a_1 - a_1 r)^2 + (a_1 r - a_1 r^2)^2 + (a_1 r^2 - a_1 r^3) + \ldots$ ***M1A1***

$= [a_1(1-r)]^2 + [a_1 r(1-r)]^2 + \left[a_1 r^2(1-r)\right]^2 + \ldots + \left[a_1 r^{n-1}(1-r)\right]^2$ ***(A1)***

Note: This ***A1*** is for the expression for the last term.

$= a_1^2(1-r)^2 + a_1^2 r^2(1-r)^2 + a_1^2 r^4(1-r)^2 + \ldots + a_1^2 r^{2n-2}(1-r)^2$ ***A1***

$= a_1^2(1-r)^2(1 + r^2 + r^4 + \ldots + r^{2n-2})$ ***A1***

$= a_1^2(1-r)^2\left(\frac{1-r^{2n}}{1-r^2}\right)$ ***M1A1***

$= \frac{a_1^2(1-r)(1-r^{2n})}{1+r}$ ***AG***

[7 marks]

Examiners report

Many candidates were able to solve (a) successfully. A few candidates failed to understand the relationship between S_{2n} and S_{3n}, and hence did not obtain the correct equation. (b) was answered poorly by a large number of candidates. There was significant difficulty in forming correct general statements, and a general lack of rigor in providing justification.

16a. *[1 mark]*

Markscheme

$\left|e^{i\theta}\right| \ \ (= |\cos\theta + i\sin\theta|) = \sqrt{\cos^2\theta + \sin^2\theta} = 1$ ***M1AG***

[1 mark]

Examiners report

Parts (b) and (c) were answered fairly well by quite a few candidates. In (a) many candidates failed to write the formula for the modulus of a complex number. (c) proved inaccessible for a large number of candidates. The algebraic manipulation required and the recognition of the imaginary and real parts in order to arrive at the necessary relationship were challenging for many candidates.

16b. *[2 marks]*

Markscheme

$z = \frac{1}{3}e^{i\theta}$ ***A1***

$|z| = \left|\frac{1}{3}e^{i\theta}\right| = \frac{1}{3}$ ***A1AG***

[2 marks]

Examiners report

Parts (b) and (c) were answered fairly well by quite a few candidates. In (a) many candidates failed to write the formula for the modulus of a complex number. (c) proved inaccessible for a large number of candidates. The algebraic manipulation required and the recognition of the imaginary and real parts in order to arrive at the necessary relationship were challenging for many candidates.

16c. *[2 marks]*

Markscheme

$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}\mathrm{e}^{\mathrm{i}\theta}}$ ***(M1)A1***

[2 marks]

Examiners report

Parts (b) and (c) were answered fairly well by quite a few candidates. In (a) many candidates failed to write the formula for the modulus of a complex number. (c) proved inaccessible for a large number of candidates. The algebraic manipulation required and the recognition of the imaginary and real parts in order to arrive at the necessary relationship were challenging for many candidates.

16d. *[8 marks]*

Markscheme

EITHER

$S_\infty = \frac{1}{1-\frac{1}{3}\cos\theta-\frac{1}{3}\mathrm{i}\sin\theta}$ ***A1***

$= \frac{1-\frac{1}{3}\cos\theta+\frac{1}{3}\mathrm{i}\sin\theta}{\left(1-\frac{1}{3}\cos\theta-\frac{1}{3}\mathrm{i}\sin\theta\right)\left(1-\frac{1}{3}\cos\theta+\frac{1}{3}\mathrm{i}\sin\theta\right)}$ ***M1A1***

$= \frac{1-\frac{1}{3}\cos\theta+\frac{1}{3}\mathrm{i}\sin\theta}{\left(1-\frac{1}{3}\cos\theta\right)^2+\frac{1}{9}\sin^2\theta}$ ***A1***

$= \frac{1-\frac{1}{3}\cos\theta+\frac{1}{3}\mathrm{i}\sin\theta}{1-\frac{2}{3}\cos\theta+\frac{1}{9}}$ ***A1***

OR

$S_\infty = \frac{1}{1-\frac{1}{3}\mathrm{e}^{\mathrm{i}\theta}}$

$= \frac{1-\frac{1}{3}\mathrm{e}^{-\mathrm{i}\theta}}{\left(1-\frac{1}{3}\mathrm{e}^{\mathrm{i}\theta}\right)\left(1-\frac{1}{3}\mathrm{e}^{-\mathrm{i}\theta}\right)}$ ***M1A1***

$= \frac{1-\frac{1}{3}\mathrm{e}^{-\mathrm{i}\theta}}{1-\frac{1}{3}(\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta})+\frac{1}{9}}$ ***A1***

$= \frac{1-\frac{1}{3}\mathrm{e}^{-\mathrm{i}\theta}}{\frac{10}{9}-\frac{2}{3}\cos\theta}$ ***A1***

$= \frac{1-\frac{1}{3}(\cos\theta - \mathrm{i}\sin\theta)}{\frac{10}{9}-\frac{2}{3}\cos\theta}$ ***A1***

THEN

taking imaginary parts on both sides

$\frac{1}{3}\sin\theta + \frac{1}{9}\sin 2\theta + \ldots = \frac{\frac{1}{3}\sin\theta}{\frac{10}{9}-\frac{2}{3}\cos\theta}$ ***M1A1A1***

$= \frac{\sin\theta}{\frac{10}{9}-\frac{2}{3}\cos\theta}$

$\Rightarrow \sin\theta + \frac{1}{3}\sin 2\theta + \ldots = \frac{9\sin\theta}{10-6\cos\theta}$ ***AG***

[8 marks]

Examiners report

Parts (b) and (c) were answered fairly well by quite a few candidates. In (a) many candidates failed to write the formula for the modulus of a complex number. (c) proved inaccessible for a large number of candidates. The algebraic manipulation required and the recognition of the imaginary and real parts in order to arrive at the necessary relationship were challenging for many candidates.

17a. *[2 marks]*

Markscheme

$u_1 = 27\frac{81}{2} = \frac{27}{1-r}$ ***M1*** $r = \frac{1}{3}$ ***A1***

[2 marks]

Examiners report

Part (a) was well done by most candidates. However (b) caused difficulty to most candidates. Although a number of different approaches were seen, just a small number of candidates obtained full marks for this question.

17b. *[5 marks]*

Markscheme

$v_2 = 9 v_4 = 1 2d = -8 \Rightarrow d = -4$ ***(A1)*** $v_1 = 13$ ***(A1)***

$\frac{N}{2}(2 \times 13 - 4(N-1)) > 0$ (accept equality) ***M1***

$\frac{N}{2}(30 - 4N) > 0$

$N(15 - 2N) > 0$

$N < 7.5$ ***(M1)***

$N = 7$ ***A1***

Note: $13 + 9 + 5 + 1 - 3 - 7 - 11 > 0 \Rightarrow N = 7$ or equivalent receives full marks.

[5 marks]

Examiners report

Part (a) was well done by most candidates. However (b) caused difficulty to most candidates. Although a number of different approaches were seen, just a small number of candidates obtained full marks for this question.

18. *[17 marks]*

Markscheme

(a) $r = 1 + \mathrm{i}$ ***(A1)***

$u_4 = 3(1+\mathrm{i})^3$ ***M1***

$= -6 + 6\mathrm{i}$ ***A1***

[3 marks]

(b) $S_{20} = \frac{\left((1+\mathrm{i})^{20} - 1\right)}{\mathrm{i}}$ ***(M1)***

$= \frac{3\left((2i)^{10} - 1\right)}{\mathrm{i}}$ ***(M1)***

Note: Only one of the two ***M1***s can be implied. Other algebraic methods may be seen.

$= \frac{3\left(-2^{10} - 1\right)}{\mathrm{i}}$ ***(A1)***

$= 3\mathrm{i}\left(2^{10} + 1\right)$ ***A1***

[4 marks]

(c) (i) **METHOD 1**

$v_n = \left(3(1+\mathrm{i})^{n-1}\right)\left(3(1+\mathrm{i})^{n-1+k}\right)$ ***M1***

$9(1+\mathrm{i})^k(1+i)^{2n-2}$ ***A1***

$= 9(1+\mathrm{i})^k\left((1+i)^2\right)^{n-1} \left(= 9(1+\mathrm{i})^k(2\mathrm{i})^{n-1}\right)$

this is the general term of a geometrical sequence ***R1AG***

Notes: Do not accept the statement that the product of terms in a geometric sequence is also geometric unless justified further.

If the final expression for v_n is $9(1+\mathrm{i})^k(1+i)^{2n-2}$ award ***M1A1R0***.

METHOD 2

$\frac{v_{n+1}}{v_n} = \frac{u_{n+1}u_{n+k+1}}{u_n u_{n+k}}$ ***M1***

$= (1 + \mathrm{i})(1 + \mathrm{i})$ ***A1***

this is a constant, hence sequence is geometric ***R1AG***

Note: Do not allow methods that do not consider the general term.

(ii) $9(1 + \mathrm{i})^k$ ***A1***

(iii) common ratio is $(1 + i)^2 \ (= 2i)$ (which is independent of *k*) ***A1***

[5 marks]

(d) (i) **METHOD 1**

$w_n \left|3(1 + i)^{n-1} - 3(1 + \mathrm{i})^n\right|$ ***M1***

$= 3|1 + i|^{n-1} \, |1 - (1 + \mathrm{i})|$ ***M1***

$= 3|1 + i|^{n-1}$ ***A1***

$\left(= 3\left(\sqrt{2}\right)^{n-1}\right)$

this is the general term for a geometric sequence ***R1AG***

METHOD 2

$w_n = |u_n - (1 + \mathrm{i})u_n|$ ***M1***

$= |u_n| \, |-\mathrm{i}|$

$= |u_n|$ ***A1***

$= \left|3(1 + \mathrm{i})^{n-1}\right|$

$= 3|(1 + \mathrm{i})|^{n-1}$ ***A1***

$\left(= 3\left(\sqrt{2}\right)^{n-1}\right)$

this is the general term for a geometric sequence ***R1AG***

Note: Do not allow methods that do not consider the general term.

(ii) distance between successive points representing u_n in the complex plane forms a geometric sequence ***R1***

Note: Various possibilities but must mention distance between successive points.

[5 marks]

Total [17 marks]

Examiners report

[N/A]

19. *[7 marks]*

Markscheme

(a) $\sin x,\ \ \sin 2x$ and $4\sin x\cos^2 x$

$r = \frac{2\sin x\cos x}{\sin x} = 2\cos x$ **A1**

Note: Accept $\frac{\sin 2x}{\sin x}$.

[1 mark]

(b) **EITHER**

$|r| < 1 \Rightarrow |2\cos x| < 1$ ***M1***

OR

$-1 < r < 1 \Rightarrow -1 < 2\cos x < 1$ ***M1***

THEN

$0 < \cos x < \frac{1}{2}$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$-\frac{\pi}{2} < x < -\frac{\pi}{3}$ or $\frac{\pi}{3} < x < \frac{\pi}{2}$ ***A1A1***

[3 marks]

(c) $S_\infty = \frac{\sin x}{1-2\cos x}$ ***M1***

$S_\infty = \frac{\sin\left(\arccos\left(\frac{1}{4}\right)\right)}{1-2\cos\left(\arccos\left(\frac{1}{4}\right)\right)}$

$= \frac{\frac{\sqrt{15}}{4}}{\frac{1}{2}}$ ***A1A1***

Note: Award ***A1*** for correct numerator and ***A1*** for correct denominator.

$= \frac{\sqrt{15}}{2}$ ***AG***

[3 marks]

Total [7 marks]

Examiners report

[N/A]

20. *[6 marks]*

Markscheme

(a) (i) $n = 27$ ***(A1)***

METHOD 1

$S_{27} = \frac{14+196}{2} \times 27$ ***(M1)***

$= 2835$ ***A1***

METHOD 2

$S_{27} = \frac{27}{2}\left(2 \times 14 + 26 \times 7\right)$ ***(M1)***

$= 2835$ ***A1***

METHOD 3

$S_{27} \sum_{n=1}^{27} 7 + 7n$ ***(M1)***

$= 2835$ ***A1***

(ii) $\sum_{n=1}^{27} (7 + 7n)$ or equivalent ***A1***

Note: Accept $\sum_{n=2}^{28} 7n$

[4 marks]

(b) $\frac{n}{2}\left(2000 - 6(n-1)\right) < 0$ ***(M1)***

$n > 334.333$

$n = 335$ ***A1***

Note: Accept working with equalities.

[2 marks]

Total [6 marks]

Examiners report

[N/A]

21. *[7 marks]*

Markscheme

(a) **METHOD 1**

$a + ar = 10$ ***A1***

$a + ar + ar^2 + ar^3 = 30$ ***A1***

$a + ar = 10 \Rightarrow ar^2 + ar^3 = 10r^2$ **or** $ar^2 + ar^3 = 20$ ***M1***

$10 + 10r^2 = 30$ **or** $r^2(a + ar) = 20$ ***A1***

$\Rightarrow r^2 = 2$ ***AG***

METHOD 2

$\frac{a(1-r^2)}{1-r} = 10$ and $\frac{a(1-r^4)}{1-r} = 30$ ***M1A1***

$\Rightarrow \frac{1-r^4}{1-r^2} = 3$ ***M1***

leading to either $1 + r^2 = 3$ (or $r^4 - 3r^2 + 2 = 0$) ***A1***

$\Rightarrow r^2 = 2$ ***AG***

[4 marks]

(b) (i) $a + a\sqrt{2} = 10$

$\Rightarrow a = \frac{10}{1+\sqrt{2}}$ **or** $a = 10\left(\sqrt{2} - 1\right)$ ***A1***

(ii) $S_{10} = \frac{10}{1+\sqrt{2}}\left(\frac{\sqrt{2}^{10}-1}{\sqrt{2}-1}\right)$ $(= 10 \times 31)$ ***M1***

$= 310$ ***A1***

[3 marks]

Total [7 marks]

Examiners report

This question was invariably answered very well. Candidates showed some skill in algebraic manipulation to derive the given answer in part a). Poor attempts at part b) were a rarity, though the final mark was sometimes lost after a correctly substituted equation was seen but with little follow-up work.

22. *[7 marks]*

Markscheme

(a) **METHOD 1**

$34 = a + 3d$ and $76 = a + 9d$ ***(M1)***

$d = 7$ ***A1***

$a = 13$ ***A1***

METHOD 2

$76 = 34 + 6d$ ***(M1)***

$d = 7$ ***A1***

$34 = a + 21$

$a = 13$ ***A1***

[3 marks]

(b) $\frac{n}{2}(26 + 7(n-1)) > 5000$ ***(M1)(A1)***

$n > 36.463\ldots$ ***(A1)***

Note: Award ***M1A1A1*** for using either an equation, a graphical approach or a numerical approach.

$n = 37$ ***A1 N3***

[4 marks]

Total [7 marks]

Examiners report

Both parts were very well done. In part (a), a few candidates made a careless algebraic error when attempting to find the value of *a* or *d*.

In part (b), a few candidates attempted to find the value of *n* for which $u_n > 5000$. Some candidates used the incorrect formula $S_n = \frac{n}{2}[u_1 + (n-1)d]$. A number of candidates unnecessarily attempted to simplify S_n. Most successful candidates in part (b) adopted a graphical approach and communicated their solution effectively. A few candidates did not state their value of *n* as an integer.