SL Normal Distribution May 2008-2014 with MS

1a. *[3 marks]* The heights of a group of seven-year-old children are normally distributed with mean 117 cm and standard deviation 5 cm. A child is chosen at random from the group.

Find the probability that this child is taller than 122.5 cm.

1b. *[3 marks]* The heights of a group of seven-year-old children are normally distributed with mean 117 cm and standard deviation 5 cm. A child is chosen at random from the group.

The probability that this child is shorter than $k \text{ cm}$ is 0.65. Find the value of k .

2a. *[4 marks]*

A box contains a large number of biscuits. The weights of biscuits are normally distributed with mean 7g and standard deviation 0.5g .

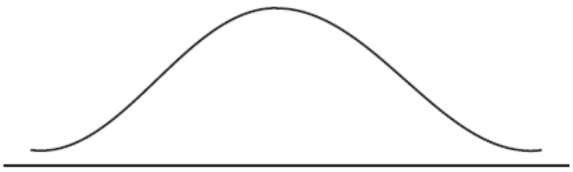
One biscuit is chosen at random from the box. Find the probability that this biscuit

(i) weighs less than 8g ;

(ii) weighs between 6g and 8g .

2b. *[5 marks]* Five percent of the biscuits in the box weigh less than *d* grams.

(i) Copy and complete the following normal distribution diagram, to represent this information, by indicating *d*, and shading the appropriate region.



(ii) Find the value of *d*.

2c. *[4 marks]* The weights of biscuits in another box are normally distributed with mean μ and standard deviation 0.5g . It is known that 20% of the biscuits in this second box weight less than 5g .

Find the value of μ .

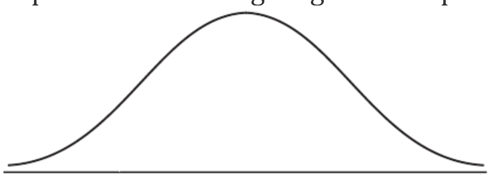
3a. *[2 marks]* The heights of certain plants are normally distributed. The plants are classified into three categories.

The shortest 12.92% are in category A.

The tallest 10.38% are in category C.

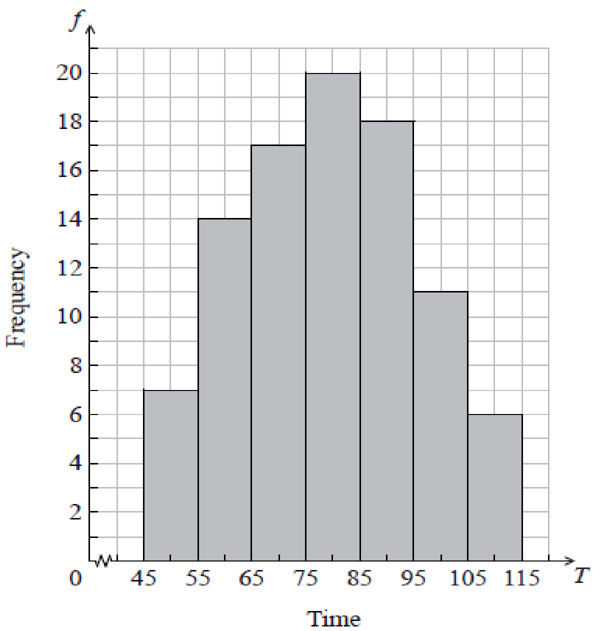
All the other plants are in category B with heights between $r \text{ cm}$ and $t \text{ cm}$.

Complete the following diagram to represent this information.



3b. *[5 marks]* Given that the mean height is 6.84 cm and the standard deviation 0.25 cm , find the value of *r* and of *t*.

4a. *[3 marks]* The histogram below shows the time *T* seconds taken by 93 children to solve a puzzle.



The following is the frequency distribution for *T* .

Time	$45 \le T < 55$	$55 \le T < 65$	$65 \le T < 75$	$75 \le T < 85$	$85 \le T < 95$	$95 \le T < 105$	$105 \le T < 115$
Frequency	7	14	p	20	18	q	6

(i) Write down the value of p and of q .
(ii) Write down the median class.

4b. *[2 marks]*
A child is selected at random. Find the probability that the child takes less than 95 seconds to solve the puzzle.

4c. *[2 marks]*
Consider the class interval $45 \le T < 55$.
(i) Write down the interval width.
(ii) Write down the mid-interval value.

4d. *[4 marks]*
Hence find an estimate for the
(i) mean;
(ii) standard deviation.

4e. *[2 marks]*
John assumes that T is normally distributed and uses this to estimate the probability that a child takes less than 95 seconds to solve the puzzle.
Find John's estimate.

5a. *[2 marks]*
A random variable X is distributed normally with mean 450 and standard deviation 20.
Find $\mathrm{P}(X \le 475)$.

5b. *[4 marks]*
Given that $\mathrm{P}(X > a) = 0.27$, find a.

6a. *[2 marks]*
A van can take either Route A or Route B for a particular journey.
If Route A is taken, the journey time may be assumed to be normally distributed with mean 46 minutes and a standard deviation 10 minutes.
If Route B is taken, the journey time may be assumed to be normally distributed with mean μ minutes and standard deviation 12 minutes.
For Route A, find the probability that the journey takes **more** than 60 minutes.

6b. *[3 marks]*
For Route B, the probability that the journey takes **less** than 60 minutes is 0.85.
Find the value of μ .

6c. *[3 marks]*
The van sets out at 06:00 and needs to arrive before 07:00.
(i) Which route should it take?
(ii) Justify your answer.

6d. *[5 marks]*
On five consecutive days the van sets out at 06:00 and takes Route B. Find the probability that
(i) it arrives before 07:00 on all five days;
(ii) it arrives before 07:00 on at least three days.

7a. *[2 marks]*
The weights of players in a sports league are normally distributed with a mean of 76.6 kg, (correct to three significant figures). It is known that 80% of the players have weights between 68 kg and 82 kg. The probability that a player weighs less than 68 kg is 0.05.
Find the probability that a player weighs more than 82 kg.

7b. *[4 marks]*
(i) Write down the standardized value, z, for 68 kg.
(ii) Hence, find the standard deviation of weights.

7c. *[5 marks]*
To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean.
(i) Find the set of all possible weights of players that take part in the tournament.
(ii) A player is selected at random. Find the probability that the player takes part in the tournament.

7d. *[4 marks]*
Of the players in the league, 25% are women. Of the women, 70% take part in the tournament.
Given that a player selected at random takes part in the tournament, find the probability that the selected player is a woman.

8a. *[3 marks]*

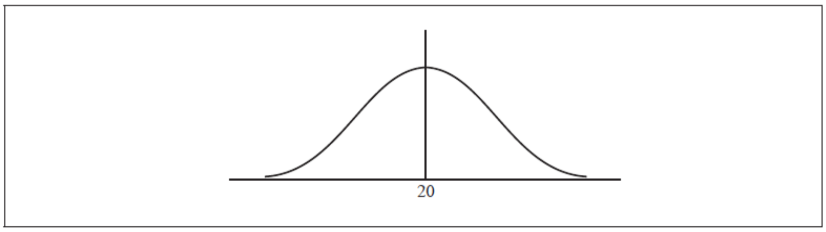
A random variable X is distributed normally with a mean of 20 and variance 9.

Find $\mathrm{P}(X \le 24.5)$.

8b. *[5 marks]*

Let $\mathrm{P}(X \le k) = 0.85$.

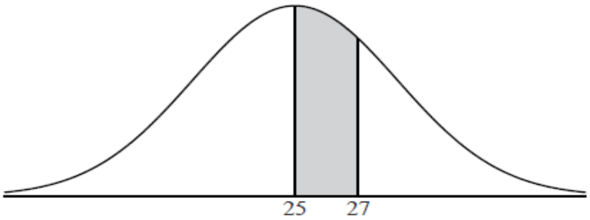
(i) Represent this information on the following diagram.



(ii) Find the value of k.

9a. *[2 marks]*

Let the random variable X be normally distributed with mean 25, as shown in the following diagram.



The shaded region between 25 and 27 represents 30% of the distribution.

Find $\mathrm{P}(X > 27)$.

9b. *[5 marks]*

Find the standard deviation of X.

10. *[7 marks]*

A random variable X is normally distributed with $\mu = 150$ and $\sigma = 10$.

Find the interquartile range of X.

11a. *[3 marks]*

The random variable X is normally distributed with mean 20 and standard deviation 5.

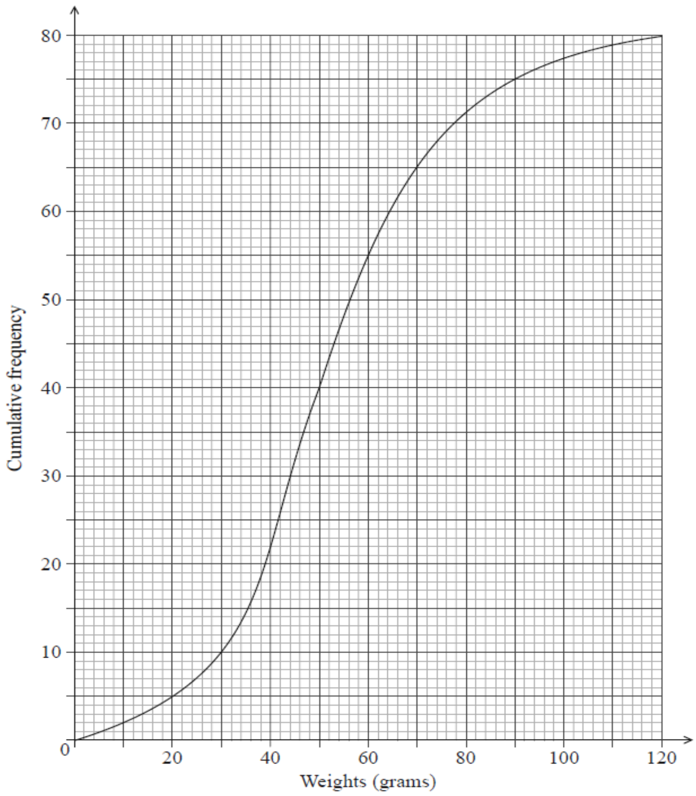
Find $\mathrm{P}(X \le 22.9)$.

11b. *[3 marks]*

Given that $\mathrm{P}(X < k) = 0.55$, find the value of k.

12a. *[1 mark]*

The weights in grams of 80 rats are shown in the following cumulative frequency diagram.



*Do **NOT** write solutions on this page.*

Write down the median weight of the rats.

12b. *[3 marks]*

Find the percentage of rats that weigh 70 grams or less.

12c. *[2 marks]*

The same data is presented in the following table.

Weights w **grams**	$0 \leqslant w \leqslant 30$	$30 < w \leqslant 60$	$60 < w \leqslant 90$	$90 < w \leqslant 120$
Frequency	p	45	q	5

Write down the value of p.

12d. *[2 marks]*

The same data is presented in the following table.

Weights w **grams**	$0 \leqslant w \leqslant 30$	$30 < w \leqslant 60$	$60 < w \leqslant 90$	$90 < w \leqslant 120$
Frequency	p	45	q	5

Find the value of q.

12e. *[3 marks]*

The same data is presented in the following table.

Weights w **grams**	$0 \leqslant w \leqslant 30$	$30 < w \leqslant 60$	$60 < w \leqslant 90$	$90 < w \leqslant 120$
Frequency	p	45	q	5

Use the values from the table to estimate the mean and standard deviation of the weights.

12f. *[2 marks]*

Assume that the weights of these rats are normally distributed with the mean and standard deviation estimated in part (c).

Find the percentage of rats that weigh 70 grams or less.

12g. *[3 marks]*

Assume that the weights of these rats are normally distributed with the mean and standard deviation estimated in part (c).

A sample of five rats is chosen at random. Find the probability that at most three rats weigh 70 grams or less.

13a. *[3 marks]*

A forest has a large number of tall trees. The heights of the trees are normally distributed with a mean of 53 metres and a standard deviation of 8 metres. Trees are classified as giant trees if they are more than 60 metres tall.

A tree is selected at random from the forest.

Find the probability that this tree is a giant.

13b. *[3 marks]*

A tree is selected at random from the forest.

Given that this tree is a giant, find the probability that it is taller than 70 metres.

13c. *[2 marks]*

Two trees are selected at random. Find the probability that they are both giants.

13d. *[3 marks]*

100 trees are selected at random.

Find the expected number of these trees that are giants.

13e. *[3 marks]*

100 trees are selected at random.

Find the probability that at least 25 of these trees are giants.

SL Normal Distribution May 2008-2014

1a. *[3 marks]*

Markscheme

evidence of appropriate method ***(M1)***

e.g. $z = \frac{122.5-117}{5}$, sketch of normal curve showing mean and 122.5, 1.1

$\mathrm{P}(Z < 1.1) = 0.8643$ ***(A1)***

0.135666

$\mathrm{P}(\mathrm{H} > 122.5) = 0.136$ ***A1 N3***

[3 marks]

Examiners report

There were many completely successful attempts at this question, with good use of formulae and calculator features.

1b. *[3 marks]*

Markscheme

$z = 0.3853$ ***(A1)***

set up equation ***(M1)***

e.g. $\frac{X-117}{5} = 0.3853$, sketch

$k = 118.926602$

$k = 199$ ***A1 N3***

[3 marks]

Examiners report

There were many completely successful attempts at this question, with good use of formulae and calculator features.

However, in part (b) some candidates did not recognize the need to find the standardized value and set their equation equal to the probability given in the question, thus earning only one mark.

2a. *[4 marks]*

Markscheme

$X \sim \mathrm{N}(7, 0.5^2)$

(i) $z = 2$ ***(M1)***

$\mathrm{P}(X < 8) = \mathrm{P}(Z < 2) = 0.977$ ***A1 N2***

(ii) evidence of appropriate approach ***(M1)***

e.g. symmetry, $z = -2$

$\mathrm{P}(6 < X < 8) = 0.954$ (tables 0.955) ***A1 N2***

Note: Award ***M1A1(AP)*** if candidates refer to 2 standard deviations from the mean, leading to 0.95.

[4 marks]

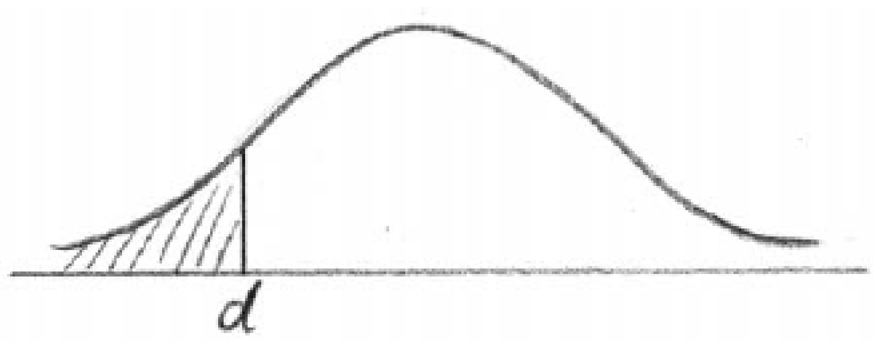
Examiners report

Those that understood the normal distribution did well on parts (a) and (bi).

2b. *[5 marks]*

Markscheme

(i)



A1A1 N2

Note: Award ***A1*** for *d* to the left of the mean, ***A1*** for area to the left of *d* shaded.

(ii) $z = -1.645$ ***(A1)***

$\frac{d-7}{0.5} = -1.645$ ***(M1)***

$d = 6.18$ ***A1 N3***

[5 marks]

Examiners report

Those that understood the normal distribution did well on parts (a) and (bi). Parts (bii) and (c) proved to be a little more difficult. In particular, in part (bii) the z-score was incorrectly set equal to 0.05 and in part (c), 0.2 was used instead of the *z*-score. For those who had a good grasp of the concept of normal distributions the entire question was quite accessible and full marks were gained.

2c. *[4 marks]*

Markscheme

$Y \sim \mathrm{N}(\mu, 0.5^2)$

$\mathrm{P}(Y < 5) = 0.2$ ***(M1)***

$z = -0.84162\ldots$ ***A1***

$\frac{5-\mu}{0.5} = -0.8416$ ***(M1)***

$\mu = 5.42$ ***A1 N3***

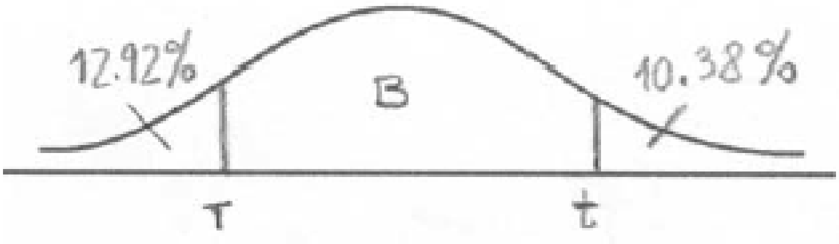
[4 marks]

Examiners report

Those that understood the normal distribution did well on parts (a) and (bi). Parts (bii) and (c) proved to be a little more difficult. In particular, in part (bii) the *z*-score was incorrectly set equal to 0.05 and in part (c), 0.2 was used instead of the *z*-score. For those who had a good grasp of the concept of normal distributions the entire question was quite accessible and full marks were gained.

3a. *[2 marks]*

Markscheme



A1A1 N2

Notes: Award ***A1*** for three regions (may be shown by lines or shading), ***A1*** for clear labelling of two regions (may be shown by percentages or categories). *r* and *t* need not be labelled, but if they are, they may be interchanged.

[2 marks]

Examiners report

Many candidates shaded or otherwise correctly labelled the appropriate regions in the normal curve.

3b. *[5 marks]*

Markscheme

METHOD 1

$\mathrm{P}(X < r) = 0.1292$ ***(A1)***

$r = 6.56$ ***A1 N2***

$1 - 0.1038$ (= 0.8962) (may be seen later) ***A1***

$\mathrm{P}(X < t) = 0.8962$ ***(A1)***

$t = 7.16$ ***A1 N2***

METHOD 2

finding *z*-values $-1.130\ldots, 1.260\ldots$ ***A1A1***

evidence of setting up one standardised equation ***(M1)***

e.g. $\frac{r-6.84}{0.25} = -1.13\ldots$, $t = 1.260 \times 0.25 + 6.84$

$r = 6.56$, $t = 7.16$ ***A1A1 N2N2***

[5 marks]

Examiners report

Although many candidates shaded or otherwise correctly labelled the appropriate regions in the normal curve, far fewer could apply techniques of normal probabilities to achieve correct results in part (b). Many set the standardized formula equal to the probabilities instead of the appropriate *z*-scores, which can be found either by the use of tables or the GDC. Others simply left this part blank, which suggests a lack of preparation for such “inverse” types of questions in a normal distribution.

4a. *[3 marks]*

Markscheme

(i) $p = 17$, $q = 11$ ***A1A1 N2***

(ii) $75 \le T < 85$ ***A1 N1***

[3 marks]

Examiners report

Parts (a) and (b) were generally well done. The terms "median" and "median class" were often confused.

4b. *[2 marks]*

Markscheme

evidence of valid approach ***(M1)***

e.g. adding frequencies

$\frac{76}{93} = 0.8172043\ldots$

$\mathrm{P}(T < 95) = \frac{76}{93} = 0.817$ ***A1 N2***

[2 marks]

Examiners report

Parts (a) and (b) were generally well done. The terms "median" and "median class" were often confused.

4c. *[2 marks]*

Markscheme

(i) 10 ***A1 N1***

(ii) 50 ***A1 N1***

[2 marks]

Examiners report

In part (c) some candidates had problems with the term "interval width" and there were some rather interesting mid-interval values noted.

4d. *[4 marks]*

Markscheme

(i) evidence of approach using mid-interval values (may be seen in part (ii)) ***(M1)***

79.1397849

$\bar{x} = 79.1$ ***A2 N3***

(ii) 16.4386061

$\sigma = 16.4$ ***A1 N1***

[4 marks]

Examiners report

In part (d), candidates often ignored the "hence" command and estimated values from the graph rather than from the information in part (c).

4e. *[2 marks]*

Markscheme

e.g. standardizing, $z = 0.9648\ldots$

0.8326812

$\mathrm{P}(T < 95) = 0.833$ ***A1 N2***

[2 marks]

Examiners report

Those who correctly obtained the mean and standard deviation had little difficulty with part (e) although candidates often used unfamiliar calculator notation as their working or used the mid-interval value as the mean of the distribution.

5a. *[2 marks]*

Markscheme

evidence of attempt to find $\mathrm{P}(X \le 475)$ ***(M1)***

e.g. $\mathrm{P}(Z \le 1.25)$

$\mathrm{P}(X \le 475) = 0.894$ ***A1 N2***

[2 marks]

Examiners report

It remains very clear that some centres still do not give appropriate attention to the normal distribution. This is a major cause for concern. Most candidates had been taught the topic but many had difficulty understanding the difference between z, $F(z)$, a and x . Very little working was shown which demonstrated understanding. Although the GDC was used extensively, candidates often worked with the wrong tail and did not write their answers correct to 3 significant figures.

5b. *[4 marks]*

Markscheme

evidence of using the complement ***(M1)***

e.g. 0.73, $1-p$
$z=0.6128$ ***(A1)***
setting up equation ***(M1)***
e.g. $\frac{a-450}{20}=0.6128$
$a=462$ ***A1 N3***
[4 marks]
Examiners report
It remains very clear that some centres still do not give appropriate attention to the normal distribution. This is a major cause for concern. Most candidates had been taught the topic but many had difficulty understanding the difference between z, $F(z)$, a and x . Very little working was shown which demonstrated understanding. Although the GDC was used extensively, candidates often worked with the wrong tail and did not write their answers correct to 3 significant figures. Many candidates had trouble with part (b), a majority never found the complement, instead using their GDCs to calculate the result, which many times was finding a for $P(X \leqslant a)=0.27$ instead of for $P(X \geqslant a)=0.27$. Many others substituted the values of 0.27 or 0.73 into the equation, instead of the z-scores.
6a. *[2 marks]*
Markscheme
$A \sim N(46, 10^2)$ $B \sim N(\mu, 12^2)$
$\mathrm{P}(A>60)=0.0808$ ***A2 N2***
[2 marks]
Examiners report
A significant number of students clearly understood what was asked in part (a) and used the GDC to find the result.
6b. *[3 marks]*
Markscheme
correct approach ***(A1)***
e.g. $\mathrm{P}\left(Z<\frac{60-\mu}{12}\right)=0.85$, sketch
$\frac{60-\mu}{12}=1.036\ldots$ ***(A1)***
$\mu=47.6$ ***A1 N2***
[3 marks]
Examiners report
In part (b), many candidates set the standardized formula equal to the probability (0.85), instead of using the corresponding *z*-score. Other candidates used the solver on their GDC with the inverse norm function.
6c. *[3 marks]*
Markscheme
(i) route A ***A1 N1***
(ii) **METHOD 1**
$\mathrm{P}(A<60)=1-0.0808=0.9192$ ***A1***
valid reason ***R1***
e.g. probability of *A* getting there on time is greater than probability of *B*
$0.9192>0.85$ ***N2***
METHOD 2
$\mathrm{P}(B>60)=1-0.85=0.15$ ***A1***
valid reason ***R1***
e.g. probability of *A* getting there late is less than probability of *B*
$0.0808<0.15$ ***N2***
[3 marks]
Examiners report
A common incorrect approach in part (c) was to attempt to use the means and standard deviations for justification, although many candidates successfully considered probabilities.
6d. *[5 marks]*
Markscheme
(i) let *X* be the number of days when the van arrives before 07:00
$\mathrm{P}(X=5)=(0.85)^5$ ***(A1)***

$= 0.444$ ***A1 N2***

(ii) **METHOD 1**

evidence of adding correct probabilities ***(M1)***

e.g. $\mathrm{P}(X \geq 3) = \mathrm{P}(X = 3) + \mathrm{P}(X = 4) + \mathrm{P}(X = 5)$

correct values $0.1382 + 0.3915 + 0.4437$ ***(A1)***

$\mathrm{P}(X \geq 3) = 0.973$ ***A1 N3***

METHOD 2

evidence of using the complement ***(M1)***

e.g. $\mathrm{P}(X \geq 3) = 1 - \mathrm{P}(X \leq 2)$, $1 - p$

correct values $1 - 0.02661$ ***(A1)***

$\mathrm{P}(X \geq 3) = 0.973$ ***A1 N3***

[5 marks]

Examiners report

A pleasing number of candidates recognized the binomial probability and made progress on part (d).

7a. *[2 marks]*

Markscheme

evidence of appropriate approach ***(M1)***

e.g. $1 - 0.85$, diagram showing values in a normal curve

$\mathrm{P}(w \geq 82) = 0.15$ ***A1 N2***

[2 marks]

Examiners report

This question was quite accessible to those candidates in centres where this topic is given the attention that it deserves. Most candidates handled part (a) well using the basic properties of a normal distribution.

7b. *[4 marks]*

Markscheme

(i) $z = -1.64$ ***A1 N1***

(ii) evidence of appropriate approach ***(M1)***

e.g. $-1.64 = \frac{x-\mu}{\sigma}$, $\frac{68-76.6}{\sigma}$

correct substitution ***A1***

e.g. $-1.64 = \frac{68-76.6}{\sigma}$

$\sigma = 5.23$ ***A1 N1***

[4 marks]

Examiners report

In part (b) (i), candidates often confused the *z*-score with the area in the table which led to a standard deviation that was less than zero in part (b) (ii). At this point, candidates “fudged” results in order to continue with the remaining parts of the question. In (b) (ii), the “hence” command was used expecting candidates to use the results of (b) (i) to find a standard deviation of 4.86. Unfortunately, many decided to use their answers and the information from part (a) resulting in quite a different standard deviation of 5.79. Recognizing the inconsistency in the question, full marks were awarded for this approach, as well as full follow-through in subsequent parts of the question.

7c. *[5 marks]*

Markscheme

(i) $68.8 \leq \text{weight} \leq 84.4$ ***A1A1A1 N3***

Note: Award ***A1*** for 68.8, ***A1*** for 84.4, ***A1*** for giving answer as an interval.

(ii) evidence of appropriate approach ***(M1)***

e.g. $\mathrm{P}(-1.5 \leq z \leq 1.5)$, $\mathrm{P}(68.76 < y < 84.44)$

$\mathrm{P}(\text{qualify}) = 0.866$ ***A1 N2***

[5 marks]

Examiners report

Candidates could obtain full marks easily in part (c) with little understanding of a normal distribution but they often confused *z*-scores with data values, adding and subtracting 1.5 from the mean of 76.

7d. *[4 marks]*

Markscheme

recognizing conditional probability ***(M1)*** e.g. $\mathrm{P}(A|B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

$\mathrm{P}(\text{woman and qualify}) = 0.25 \times 0.7$ ***(A1)***

$\mathrm{P}(\text{woman}|\text{qualify}) = \frac{0.25 \times 0.7}{0.866}$ ***A1***

$\mathrm{P}(\text{woman}|\text{qualify}) = 0.202$ ***A1***

[4 marks]

Examiners report

In part (d), few recognized the conditional nature of the question and only determined the probability that a woman qualifies **and** takes part in the tournament.

8a. *[3 marks]*

Markscheme

$\sigma = 3$ ***(A1)***

evidence of attempt to find $\mathrm{P}(X \le 24.5)$ ***(M1)***

e.g. $z = 1.5$, $\frac{24.5-20}{3}$

$\mathrm{P}(X \le 24.5) = 0.933$ ***A1 N3***

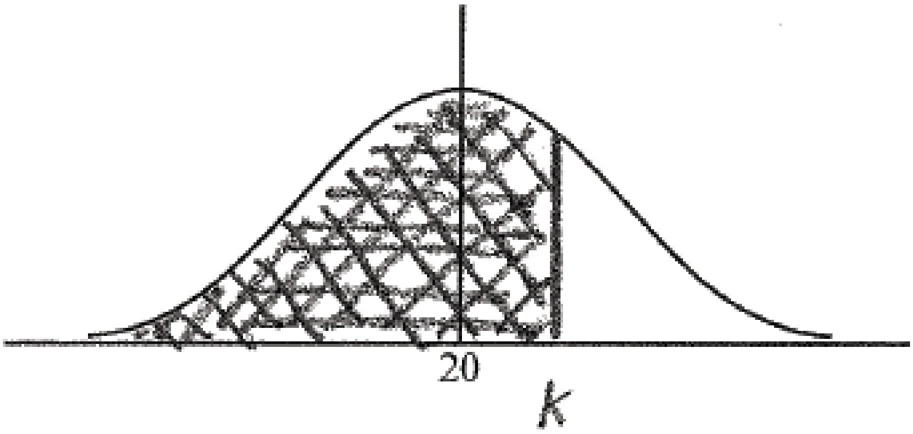
[3 marks]

Examiners report

This question clearly demonstrated that some centres are still not giving adequate treatment to this topic. A great many candidates neglected to find the standard deviation and used the variance throughout. More still did not leave their answers to the required accuracy. Ignoring the use of the variance, responses to part (a) demonstrated that most candidates were comfortable finding the required probability using their calculator or setting up a suitable standardized equation.

8b. *[5 marks]*

Markscheme



A1A1 N2

Note: Award ***A1*** with shading that clearly extends to right of the mean, ***A1*** for any correct label, either *k*, area or their value of *k*.

(ii) $z = 1.03(64338)$ ***(A1)***

attempt to set up an equation ***(M1)***

e.g. $\frac{k-20}{3} = 1.0364$, $\frac{k-20}{3} = 0.85$

$k = 23.1$ ***A1 N3***

[5 marks]

Examiners report

This question clearly demonstrated that some centres are still not giving adequate treatment to this topic. A great many candidates neglected to find the standard deviation and used the variance throughout. More still did not leave their answers to the required accuracy. Ignoring the use of the variance, responses to part (a) demonstrated that most candidates were comfortable finding the required probability using their calculator or setting up a suitable standardized equation. In part (b) (i), the sketch was often poorly shaded or incorrectly labelled. In (b) (ii), candidates frequently confused the *z*-score with the given probability of 0.85. Calculator approaches were more successful than working by hand but candidates should remember to avoid the use of calculator notation in their working, as it is not correct mathematical notation.

9a. *[2 marks]*

Markscheme

symmetry of normal curve ***(M1)***

e.g. $\mathrm{P}(X < 25) = 0.5$

$\mathrm{P}(X > 27) = 0.2$ ***A1 N2***

[2 marks]

Examiners report

This question proved challenging for many candidates. A surprising number did not use the symmetry of the normal curve to find the probability required in (a). While many students were able to set up a standardized equation in (b), far fewer were able to use the complement to find the correct *z*-score. Others used 0.8 as the *z*-score. A common confusion when approaching parts (a) and (b) was whether to use a probability or a *z*-score. Additionally, many candidates seemed unsure of appropriate notation on this problem which would have allowed them to better demonstrate their method.

9b. *[5 marks]*

Markscheme

METHOD 1

finding standardized value ***(A1)***

e.g. $\frac{27-25}{\sigma}$

evidence of complement ***(M1)***

e.g. $1 - p$, $\mathrm{P}(X < 27)$, 0.8

finding *z*-score ***(A1)***

e.g. $z = 0.84\ldots$

attempt to set up equation involving the standardized value ***M1***

e.g. $0.84 = \frac{27-25}{\sigma}$, $0.84 = \frac{X-\mu}{\sigma}$

$\sigma = 2.38$ ***A1 N3***

METHOD 2

set up using normal CDF function and probability ***(M1)***

e.g. $\mathrm{P}(25 < X < 27) = 0.3$, $\mathrm{P}(X < 27) = 0.8$

correct equation ***A2***

e.g. $\mathrm{P}(25 < X < 27) = 0.3$, $\mathrm{P}(X > 27) = 0.2$

attempt to solve the equation using GDC ***(M1)***

e.g. solver, graph, trial and error (more than two trials must be shown)

$\sigma = 2.38$ ***A1 N3***

[5 marks]

Examiners report

This question proved challenging for many candidates. A surprising number did not use the symmetry of the normal curve to find the probability required in (a). While many students were able to set up a standardized equation in (b), far fewer were able to use the complement to find the correct *z*-score. Others used 0.8 as the *z*-score. A common confusion when approaching parts (a) and (b) was whether to use a probability or a *z*-score. Additionally, many candidates seemed unsure of appropriate notation on this problem which would have allowed them to better demonstrate their method.

10. *[7 marks]*

Markscheme

recognizing one quartile probability (may be seen in a sketch) ***(M1)***

eg $\mathrm{P}(X < Q_3) = 0.75$, 0.25

finding standardized value for either quartile ***(A1)***

eg $z = 0.67448\ldots$, $z = -0.67448\ldots$

attempt to set up equation (must be with z-values) ***(M1)***

eg $0.67 = \frac{Q_3-150}{10}$, $-0.67448 = \frac{x-150}{10}$

one correct quartile

eg $Q_3 = 156.74\ldots$, $Q_1 = 143.25\ldots$

correct working ***(A1)***

eg other correct quartile, $Q_3 - \mu = 6.744\ldots$

valid approach for IQR (seen anywhere) ***(A1)***

eg $Q_3 - Q_1$, $2(Q_3 - \mu)$

IQR $= 13.5$ ***A1 N4***

[7 marks]

Examiners report

This was an accessible problem that created difficulties for candidates. Although they recognized and often wrote down a formula for IQR, most did not understand the conceptual nature of the first and third quartiles. Those who did could solve the problem effectively using their GDC in relatively few steps. Candidates that were able to start this question often drew the normal curve and gave quartile values at 140 and 160. This generally led to a solution which while wrong, was also clearly inadequate for the indicated 7 marks.

11a. *[3 marks]*

Markscheme

evidence of appropriate approach ***(M1)***

eg $z = \frac{22.9-20}{5}$

$z = 0.58$ ***(A1)***

$\mathrm{P}(X \le 22.9) = 0.719$ ***A1 N3***

[3 marks]

Examiners report

The normal distribution was handled better than in previous years with many candidates successful in both parts and very few blank responses. Some candidates used tables and z-scores while others used the GDC directly; the GDC approach earned full marks more often than the z-score approach.

11b. *[3 marks]*

Markscheme

z-score for 0.55 is $0.12566\ldots$ ***(A1)***

valid approach (must be with z-values) ***(M1)***

eg using inverse normal, $0.1257 = \frac{k-20}{5}$

$k = 20.6$ ***A1 N3***

[3 marks]

Examiners report

The normal distribution was handled better than in previous years with many candidates successful in both parts and very few blank responses. Some candidates used tables and z-scores while others used the GDC directly; the GDC approach earned full marks more often than the z-score approach. A common error in part (b) was to set the expression for z-score equal to the probability. Many candidates had difficulty giving answers correct to three significant figures; this was particularly an issue if no working was shown.

12a. *[1 mark]*

Markscheme

50 (g) ***A1 N1***

[2 marks]

Examiners report

[N/A]

12b. *[3 marks]*

Markscheme

65 rats weigh less than 70 grams ***(A1)***

attempt to find a percentage ***(M1)***

eg $\frac{65}{80}$, $\frac{65}{80} \times 100$

81.25 (%) (exact), 81.3 ***A1 N3***

[2 marks]

Examiners report

[N/A]

12c. *[2 marks]*

Markscheme

$p = 10$ ***A2 N2***

[2 marks]

Examiners report

[N/A]

12d. *[2 marks]*

Markscheme

subtracting to find q ***(M1)***

eg $75 - 45 - 10$

$q = 20$ ***A1 N2***

[2 marks]

Examiners report

[N/A]

12e. *[3 marks]*

Markscheme

evidence of mid-interval values ***(M1)***

eg $15, 45, 75, 105$

$\overline{x} = 52.5$ (exact), $\sigma = 22.5$ (exact) ***A1A1 N3***

[3 marks]

Examiners report

[N/A]

12f. *[2 marks]*

Markscheme

0.781650

78.2 (%) ***A2 N2***

[2 marks]

Examiners report

[N/A]

12g. *[3 marks]*

Markscheme

recognize binomial probability ***(M1)***

eg $X \sim \mathrm{B}(n,\ p)$, $\begin{pmatrix} 5 \\ r \end{pmatrix} \times 0.782^r \times 0.218^{5-r}$

valid approach ***(M1)***

eg $\mathrm{P}(X \leqslant 3)$

0.30067

0.301 ***A1 N2***

[3 marks]

Examiners report

[N/A]

13a. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg $\mathrm{P}(G) = \mathrm{P}(H > 60,\ z = 0.875, \mathrm{P}\ (H > 60) = 1 - 0.809, \mathrm{N}\ \left(53, 8^2\right)$

0.190786

$\mathrm{P}(G) = 0.191$ ***A1 N2***

[3 marks]

Examiners report

[N/A]

13b. *[3 marks]*

Markscheme

finding $\mathrm{P}(H > 70) = 0.01679$ (seen anywhere) ***(A1)***

recognizing conditional probability ***(R1)***

eg $\mathrm{P}(A\,|B), \mathrm{P}\ (H > 70\,|H > 60)$

correct working ***(A1)***

eg $\frac{0.01679}{0.191}$

0.0880209

$\mathrm{P}(X > 70\,|G) = 0.0880$ ***A1 N3***

[6 marks]

Examiners report

[N/A]

13c. *[2 marks]*

Markscheme

attempt to square their $\mathrm{P}(G)$ ***(M1)***

eg 0.191^2

0.0363996

$\mathrm{P}(\text{both } G) = 0.0364$ ***A1 N2***

[2 marks]

Examiners report

[N/A]

13d. *[3 marks]*

Markscheme

correct substitution into formula for $\mathrm{E}(X)$ ***(A1)***

eg $100(0.191)$

$\mathrm{E}(G) = 19.1\ [19.0,\ 19.1]$ ***A1 N2***

[3 marks]

Examiners report

[N/A]

13e. *[3 marks]*

Markscheme

recognizing binomial probability (may be seen in part (c)(i)) ***(R1)***

eg $X \sim \mathrm{B}(n,\ p)$

valid approach (seen anywhere) ***(M1)***

eg $\mathrm{P}(X \geqslant 25) = 1 - \mathrm{P}(X \leqslant 24),\ 1 - \mathrm{P}(X < a)$

correct working ***(A1)***

eg $\mathrm{P}(X \leqslant 24) = 0.913\ldots,\ 1 - 0.913\ldots$

0.0869002

$\mathrm{P}(X \geqslant 25) = 0.0869$ ***A1 N2***

[3 marks]

Examiners report

[N/A]
