SL Sequences 2008-2014 with MS

1a. *[2 marks]*
The first three terms of an arithmetic sequence are 5 , 6.7 , 8.4 .
Find the common difference.

1b. *[2 marks]*
The first three terms of an arithmetic sequence are 5 , 6.7 , 8.4 .
Find the 28term of the sequence.

1c. *[2 marks]*
The first three terms of an arithmetic sequence are 5 , 6.7 , 8.4 .
Find the sum of the first 28 terms.

2a. *[4 marks]*
The first term of a geometric sequence is 200 and the sum of the first four terms is 324.8.
Find the common ratio.

2b. *[2 marks]*
The first term of a geometric sequence is 200 and the sum of the first four terms is 324.8.
Find the tenth term.

3a. *[3 marks]*
The first three terms of an arithmetic sequence are 36, 40, 44,....
(i) Write down the value of *d* .
(ii) Find u_8 .

3b. *[3 marks]*
(i) Show that $S_n = 2n^2 + 34n$.
(ii) Hence, write down the value of S_{14} .

4a. *[4 marks]*
Consider an infinite geometric sequence with $u_1 = 40$ and $r = \frac{1}{2}$.
(i) Find u_4 .
(ii) Find the sum of the infinite sequence.

4b. *[5 marks]*
Consider an arithmetic sequence with *n* terms, with first term (-36) and eighth term (-8) .
(i) Find the common difference.
(ii) Show that $S_n = 2n^2 - 38n$.

4c. *[5 marks]*
The sum of the infinite geometric sequence is equal to twice the sum of the arithmetic sequence. Find *n* .

5a. *[2 marks]*
In an arithmetic sequence, $u_1 = 2$ and $u_3 = 8$.
Find *d* .

5b. *[2 marks]*
Find u_{20} .

5c. *[2 marks]*
Find S_{20} .

6a. *[7 marks]*
An arithmetic sequence is given by 5, 8, 11, ...
(a) Write down the value of d .
(b) Find
(i) u_{100} ;
(ii) S_{100} .
(c) Given that $u_n = 1502$, find the value of n .

6b. *[1 mark]*
Write down the value of d .

6c. *[4 marks]*
Find
(i) u_{100} ;
(ii) S_{100} .

6d. *[2 marks]*
Given that $u_n = 1502$, find the value of n .

7. *[6 marks]*
The sum of the first three terms of a geometric sequence is 62.755, and the sum of the infinite sequence is 440. Find the common ratio.

8a. *[2 marks]*

In an arithmetic sequence, the third term is 10 and the fifth term is 16.
Find the common difference.

8b. *[2 marks]*

Find the first term.

8c. *[3 marks]*

Find the sum of the first 20 terms of the sequence.

9a. *[4 marks]*

The sides of a square are 16 cm in length. The midpoints of the sides of this square are joined to form a new square and four triangles (diagram 1). The process is repeated twice, as shown in diagrams 2 and 3.

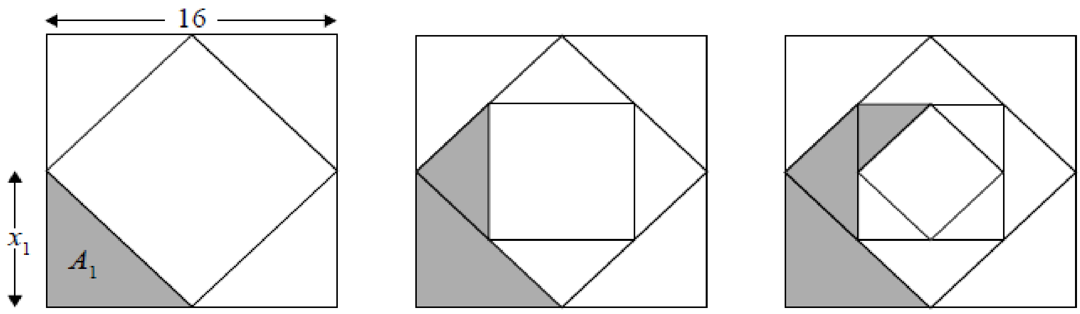

diagram 1 diagram 2 diagram 3

Let x_n denote the length of one of the equal sides of each new triangle.
Let A_n denote the area of each new triangle.
The following table gives the values of x_n and A_n, for $1 \leqslant n \leqslant 3$. **Copy** and complete the table. *(Do **not** write on this page.)*

n	1	2	3
x_n	8		4
A_n	32	16	

9b. *[4 marks]*

The process described above is repeated. Find A_6.

9c. *[7 marks]*

Consider an initial square of side length k cm. The process described above is repeated indefinitely.
The total area of the shaded regions is kc m^2. Find the value of k.

10a. *[4 marks]*

The sums of the terms of a sequence follow the pattern
$S_1 = 1 + k,\ S_2 = 5 + 3k,\ S_3 = 12 + 7k,\ S_4 = 22 + 15k,\ \ldots,$ where $k \in \mathbb{Z}$.
Given that $u_1 = 1 + k$, find $u_2,\ u_3$ and u_4.

10b. *[4 marks]*

Find a general expression for u_n.

11a. *[2 marks]*

The first three terms of a infinite geometric sequence are $m - 1,\ 6,\ m + 4$, where $m \in \mathbb{Z}$.
Write down an expression for the common ratio, r.

11b. *[2 marks]*

Hence, show that m satisfies the equation $m^2 + 3m - 40 = 0$.

11c. *[3 marks]*

Find the two possible values of m.

11d. *[3 marks]*

Find the possible values of r.

11e. *[3 marks]*

The sequence has a finite sum.
State which value of r leads to this sum **and** justify your answer.

11f. *[3 marks]*

The sequence has a finite sum.
Calculate the sum of the sequence.

SL Sequences 2008-2014 MS

1a. *[2 marks]*

Markscheme

valid method ***(M1)***

e.g. subtracting terms, using sequence formula

$d = 1.7$ ***A1 N2***

[2 marks]

Examiners report

Most candidates performed well on this question. A few were confused between the term number and the value of a term.

1b. *[2 marks]*

Markscheme

correct substitution into term formula ***(A1)***

e.g. $5 + 27(1.7)$

28 term is 50.9 (exact) ***A1 N2***

[2 marks]

Examiners report

Most candidates performed well on this question. A few were confused between the term number and the value of a term.

1c. *[2 marks]*

Markscheme

correct substitution into sum formula ***(A1)***

e.g. $S_{28} = \frac{28}{2}\left(2(5) + 27(1.7)\right)$, $\frac{28}{2}\left(5 + 50.9\right)$

$S_{28} = 782.6$ (exact) $[782, 783]$ ***A1 N2***

[2 marks]

Examiners report

Most candidates performed well on this question. A few were confused between the term number and the value of a term.

2a. *[4 marks]*

Markscheme

correct substitution into sum of a geometric sequence ***(A1)***

e.g. $200\left(\frac{1-r^4}{1-r}\right)$, $200 + 200r + 200r^2 + 200r^3$

attempt to set up an equation involving a sum and 324.8 ***M1***

e.g. $200\left(\frac{1-r^4}{1-r}\right) = 324.8$, $200 + 200r + 200r^2 + 200r^3 = 324.8$

$r = 0.4$ (exact) ***A2 N3***

[4 marks]

Examiners report

In part (a), although most candidates substituted correctly into the formula for the sum of a geometric series and set it equal to 324.8, some used the formula for the sum to infinity and a few the formula for the sum of an arithmetic series. The overwhelming error made was in attempting to solve the equation algebraically and getting nowhere, or getting a wrong answer. The great majority did not recognize the need to use the GDC to find the value of *r*.

2b. *[2 marks]*

Markscheme

correct substitution into formula ***A1***

e.g. $u_{10} = 200 \times 0.4^9$

$u_{10} = 0.0524288$ (exact), 0.0524 ***A1 N1***

[2 marks]

Examiners report

In part (b) many did not obtain any marks since they weren't able to find an answer to part (a). Those who were able to get a value for *r* in part (a) generally went on to gain full marks in (b). However, this was one of the most common places for rounding errors to be made.

3a. *[3 marks]*

Markscheme

(i) $d = 4$ ***A1 N1***

(ii) evidence of valid approach ***(M1)***

e.g. $u_8 = 36 + 7(4)$, repeated addition of *d* from 36

$u_8 = 64$ ***A1 N2***

[3 marks]

Examiners report

The majority of candidates were successful with this question. Most had little difficulty with part (a).

3b. *[3 marks]*

Markscheme

(i) correct substitution into sum formula ***A1***

e.g. $S_n = \frac{n}{2}\left\{2(36) + (n-1)(4)\right\}$, $\frac{n}{2}\left\{72 + 4n - 4\right\}$

evidence of simplifying

e.g. $\frac{n}{2}\left\{4n + 68\right\}$ ***A1***

$S_n = 2n^2 + 34n$ ***AG N0***

(ii) 868 ***A1 N1***

[3 marks]

Examiners report

Some candidates were unable to show the required result in part (b), often substituting values for *n* rather than working with the formula for the sum of an arithmetic series.

4a. *[4 marks]*

Markscheme

(i) correct approach ***(A1)***

e.g. $u_4 = (40)\frac{1}{2}^{(4-1)}$, listing terms

$u_4 = 5$ ***A1 N2***

(ii) correct substitution into formula for infinite sum ***(A1)***

e.g. $S_\infty = \frac{40}{1-0.5}$, $S_\infty = \frac{40}{0.5}$

$S_\infty = 80$ ***A1 N2***

[4 marks]

Examiners report

Most candidates found part (a) straightforward, although a common error in (a)(ii) was to calculate 40 divided by $\frac{1}{2}$ as 20.

4b. *[5 marks]*

Markscheme

(i) attempt to set up expression for u_8 ***(M1)***

e.g. $-36 + (8-1)d$

correct working ***A1***

e.g. $-8 = -36 + (8-1)d$, $\frac{-8-(-36)}{7}$

$d = 4$ ***A1 N2***

(ii) correct substitution into formula for sum ***(A1)***

e.g. $S_n = \frac{n}{2}\left(2(-36) + (n-1)4\right)$

correct working ***A1***

e.g. $S_n = \frac{n}{2}(4n - 76)$, $-36n + 2n^2 - 2n$

$S_n = 2n^2 - 38n$ ***AG N0***

[5 marks]

Examiners report

In part (b), some candidates had difficulty with the "show that" and worked backwards from the answer given.

4c. *[5 marks]*

Markscheme

multiplying S_n (AP) by 2 or dividing *S* (infinite GP) by 2 ***(M1)***

e.g. $2S_n$, $\frac{S_\infty}{2}$, 40

evidence of substituting into $2S_n = S_\infty$ ***A1***

e.g. $2n^2 - 38n = 40$, $4n^2 - 76n - 80$ $(= 0)$

attempt to solve **their** quadratic (equation) ***(M1)***

e.g. intersection of graphs, formula

$n = 20$ ***A2 N3***

[5 marks]

Examiners report

Most candidates obtained the correct equation in part (c), although some did not reject the negative value of *n* as impossible in this context.

5a. *[2 marks]*

Markscheme

attempt to find *d* ***(M1)***

e.g. $\frac{u_3-u_1}{2}$, $8=2+2d$

$d=3$ ***A1 N2***

[2 marks]

Examiners report

This question was answered correctly by the large majority of candidates. The few mistakes seen were due to either incorrect substitution into the formula or simple arithmetic errors. Even where candidates made mistakes, they were usually able to earn full follow-through marks in the subsequent parts of the question.

5b. *[2 marks]*

Markscheme

correct substitution ***(A1)***

e.g. $u_{20} = 2+(20-1)3$, $u_{20} = 3\times 20-1$

$u_{20} = 59$ ***A1 N2***

[2 marks]

Examiners report

This question was answered correctly by the large majority of candidates. The few mistakes seen were due to either incorrect substitution into the formula or simple arithmetic errors. Even where candidates made mistakes, they were usually able to earn full follow-through marks in the subsequent parts of the question.

5c. *[2 marks]*

Markscheme

correct substitution ***(A1)***

e.g. $S_{20} = \frac{20}{2}(2+59)$, $S_{20} = \frac{20}{2}(2\times 2+19\times 3)$

$S_{20} = 610$ ***A1 N2***

[2 marks]

Examiners report

This question was answered correctly by the large majority of candidates. The few mistakes seen were due to either incorrect substitution into the formula or simple arithmetic errors. Even where candidates made mistakes, they were usually able to earn full follow-through marks in the subsequent parts of the question.

6a. *[7 marks]*

Markscheme

(a) $d=3$ ***A1 N1***

[1 mark]

(b) (i) correct substitution into term formula ***(A1)***

e.g. $u_{100} = 5+3(99)$, $5+3(100-1)$

$u_{100} = 302$ ***A1 N2***

(ii) correct substitution into sum formula ***(A1)***

eg $S_{100} = \frac{100}{2}(2(5)+99(3))$, $S_{100} = \frac{100}{2}(5+302)$

$S_{100} = 15350$ ***A1 N2***

[4 marks]

(c) correct substitution into term formula ***(A1)***

eg $1502 = 5+3(n-1)$, $1502 = 3n+2$

$n=500$ ***A1 N2***

[2 marks]

Total [7 marks]

Examiners report

The majority of candidates had little difficulty with this question. If errors were made, they were normally made out of carelessness. A very few candidates mistakenly used the formulas for geometric sequences and series.

6b. *[1 mark]*

Markscheme

$d = 3$ ***A1 N1***

[1 mark]

Examiners report

The majority of candidates had little difficulty with this question. If errors were made, they were normally made out of carelessness. A very few candidates mistakenly used the formulas for geometric sequences and series.

6c. *[4 marks]*

Markscheme

(i) correct substitution into term formula ***(A1)***

e.g. $u_{100} = 5 + 3(99)$, $5 + 3(100 - 1)$

$u_{100} = 302$ ***A1 N2***

(ii) correct substitution into sum formula ***(A1)***

eg $S_{100} = \frac{100}{2}(2(5) + 99(3))$, $S_{100} = \frac{100}{2}(5 + 302)$

$S_{100} = 15350$ ***A1 N2***

[4 marks]

Examiners report

The majority of candidates had little difficulty with this question. If errors were made, they were normally made out of carelessness. A very few candidates mistakenly used the formulas for geometric sequences and series.

6d. *[2 marks]*

Markscheme

correct substitution into term formula ***(A1)***

eg $1502 = 5 + 3(n - 1)$, $1502 = 3n + 2$

$n = 500$ ***A1 N2***

[2 marks]

Total [7 marks]

Examiners report

The majority of candidates had little difficulty with this question. If errors were made, they were normally made out of carelessness. A very few candidates mistakenly used the formulas for geometric sequences and series.

7. *[6 marks]*

Markscheme

correct substitution into sum of a geometric sequence ***A1***

eg $62.755 = u_1\left(\frac{1-r^3}{1-r}\right)$, $u_1 + u_1 r + u_1 r^2 = 62.755$

correct substitution into sum to infinity ***A1***

eg $\frac{u_1}{1-r} = 440$

attempt to eliminate one variable ***(M1)***

eg substituting $u_1 = 440(1 - r)$

correct equation in one variable ***(A1)***

eg $62.755 = 440(1 - r)\left(\frac{1-r^3}{1-r}\right)$, $440(1 - r)(1 + r + r^2) = 62.755$

evidence of attempting to solve the equation in a single variable ***(M1)***

eg sketch, setting equation equal to zero, $62.755 = 440(1 - r^3)$

$r = 0.95 = \frac{19}{20}$ ***A1 N4***

[6 marks]

Examiners report

Many candidates were able to successfully obtain two equations in two variables, but far fewer were able to correctly solve for the value of r. Some candidates had misread errors for either 440 or 62.755, with some candidates taking the French and Spanish exams mistaking the decimal comma for a thousands comma. Many candidates who attempted to solve algebraically did not cancel the $1 - r$ from both sides and ended up with a 4 degree equation that they could not solve. Some of these candidates obtained the extraneous answer of $r - 1$ as well. Some candidates used a minimum of algebra to eliminate the first term and then quickly solved the resulting equation on their GDC.

8a. *[2 marks]*

Markscheme

attempt to find d ***(M1)***

eg $\frac{16-10}{2}$, $10-2d=16-4d$, $2d=6$, $d=6$

$d=3$ ***A1 N2***

[2 marks]

Examiners report

[N/A]

8b. *[2 marks]*

Markscheme

correct approach ***(A1)***

eg $10=u_1+2\times 3$, $10-3-3$

$u_1=4$ ***A1 N2***

[2 marks]

Examiners report

[N/A]

8c. *[3 marks]*

Markscheme

correct substitution into sum or term formula ***(A1)***

eg $\frac{20}{2}(2\times 4+19\times 3)$, $u_{20}=4+19\times 3$

correct simplification ***(A1)***

eg $8+57$, $4+61$

$S_{20}=650$ ***A1 N2***

[3 marks]

Examiners report

[N/A]

9a. *[4 marks]*

Markscheme

valid method for finding side length ***(M1)***

eg $8^2+8^2=c^2$, $45-45-90$ side ratios, $8\sqrt{2}$, $\frac{1}{2}s^2=16$, $x^2+x^2=8^2$

correct working for area ***(A1)***

eg $\frac{1}{2}\times 4\times 4$

n	1	2	3
x_n	8	$\sqrt{32}$	4
A_n	32	16	8

A1A1 N2N2

[4 marks]

9b. *[4 marks]*

Markscheme

METHOD 1

recognize geometric progression for A_n ***(R1)***

eg $u_n=u_1r^{n-1}$

$r=\frac{1}{2}$ ***(A1)***

correct working ***(A1)***

eg $32\left(\frac{1}{2}\right)^5$; $4, 2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$

$A_6=1$ ***A1 N3***

METHOD 2

attempt to find x_6 ***(M1)***

eg $8\left(\frac{1}{\sqrt{2}}\right)^5$, $2\sqrt{2}$, 2, $\sqrt{2}$, $1, \ldots$

$x_6=\sqrt{2}$ ***(A1)***

correct working ***(A1)***

eg $\frac{1}{2}\left(\sqrt{2}\right)^2$

$A_6 = 1$ ***A1 N3***

[4 marks]

Examiners report

[N/A]

9c. *[7 marks]*

Markscheme

METHOD 1

recognize infinite geometric series ***(R1)***

eg $S_n = \frac{a}{1-r}\ ,\ \ |r| < 1$

area of first triangle in terms of k ***(A1)***

eg $\frac{1}{2}\left(\frac{k}{2}\right)^2$

attempt to substitute into sum of infinite geometric series (must have k) ***(M1)***

eg $\frac{\frac{1}{2}\left(\frac{k}{2}\right)^2}{1-\frac{1}{2}}\ ,\ \ \frac{k}{1-\frac{1}{2}}$

correct equation ***A1***

eg $\frac{\frac{1}{2}\left(\frac{k}{2}\right)^2}{1-\frac{1}{2}} = k,\ \ k = \frac{\frac{k^2}{8}}{\frac{1}{2}}$

correct working ***(A1)***

eg $k^2 = 4k$

valid attempt to solve **their** quadratic ***(M1)***

eg $k(k-4),\ \ k = 4 \text{ or } k = 0$

$k = 4$ ***A1 N2***

METHOD 2

recognizing that there are four sets of infinitely shaded regions with equal area ***R1***

area of original square is k^2 ***(A1)***

so total shaded area is $\frac{k^2}{4}$ ***(A1)***

correct equation $\frac{k^2}{4} = k$ ***A1***

$k^2 = 4k$ ***(A1)***

valid attempt to solve their quadratic ***(M1)***

eg $k(k-4),\ \ k = 4 \text{ or } k = 0$

$k = 4$ ***A1 N2***

[7 marks]

Examiners report

[N/A]

10a. *[4 marks]*

Markscheme

valid method ***(M1)***

eg $u_2 = S_2 - S_1,\ 1 + k + u_2 = 5 + 3k$

$u_2 = 4 + 2k,\ u_3 = 7 + 4k,\ u_4 = 10 + 8k$ ***A1A1A1 N4***

[4 marks]

Examiners report

[N/A]

10b. *[4 marks]*

Markscheme

correct AP **or** GP ***(A1)***

eg finding common difference is 3, common ratio is 2

valid approach using arithmetic **and** geometric formulas ***(M1)***

eg $1 + 3(n-1)$ **and** $r^{n-1}k$

$u_n = 3n - 2 + 2^{n-1}k$ ***A1A1 N4***

Note: Award ***A1*** for $3n-2$, ***A1*** for $2^{n-1}k$.

[4 marks]

11a. *[2 marks]*

Markscheme

correct expression for r ***A1 N1***

eg $r = \frac{6}{m-1}$, $\frac{m+4}{6}$

[2 marks]

Examiners report

[N/A]

11b. *[2 marks]*

Markscheme

correct equation ***A1***

eg $\frac{6}{m-1} = \frac{m+4}{6}$, $\frac{6}{m+4} = \frac{m-1}{6}$

correct working ***(A1)***

eg $(m+4)(m-1) = 36$

correct working ***A1***

eg $m^2 - m + 4m - 4 = 36,\ m^2 + 3m - 4 = 36$

$m^2 + 3m - 40 = 0$ ***AG N0***

[2 marks]

Examiners report

[N/A]

11c. *[3 marks]*

Markscheme

valid attempt to solve ***(M1)***

eg $(m+8)(m-5) = 0,\ m = \frac{-3 \pm \sqrt{9+4\times 40}}{2}$

$m = -8,\ m = 5$ ***A1A1 N3***

[3 marks]

Examiners report

[N/A]

11d. *[3 marks]*

Markscheme

attempt to substitute **any** value of m to find r ***(M1)***

eg $\frac{6}{-8-1}$, $\frac{5+4}{6}$

$r = \frac{3}{2}$, $r = -\frac{2}{3}$ ***A1A1 N3***

[3 marks]

Examiners report

[N/A]

11e. *[3 marks]*

Markscheme

$r = -\frac{2}{3}$ (may be seen in justification) ***A1***

valid reason ***R1 N0***

eg $|r| < 1,\ -1 < \frac{-2}{3} < 1$

Notes: Award ***R1*** for $|r| < 1$ only if ***A1*** awarded.

[2 marks]

Examiners report

[N/A]

11f. *[3 marks]*

Markscheme

finding the first term of the sequence which has $|r| < 1$ ***(A1)***

eg $-8-1,\ 6 \div \frac{-2}{3}$

$u_1 = -9$ (may be seen in formula) ***(A1)***

correct substitution of u_1 and their r into $\frac{u_1}{1-r}$, as long as $|r| < 1$ ***A1***

eg $S_\infty = \frac{-9}{1-\left(-\frac{2}{3}\right)}$, $\frac{-9}{\frac{5}{3}}$

$S_\infty = -\frac{27}{5}\ (= -5.4)$ ***A1 N3***