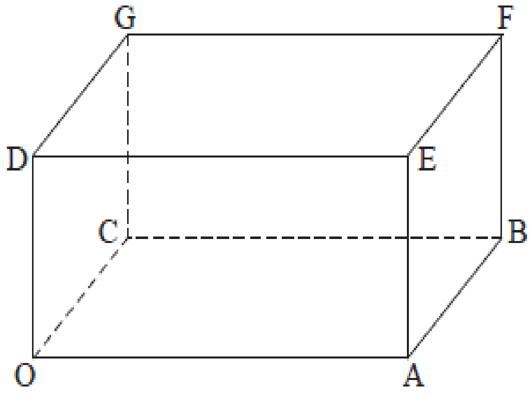
SL Vectors May 2008-14

1a. *[5 marks]*

The following diagram shows the cuboid (rectangular solid) OABCDEFG, where O is the origin, and $\overrightarrow{\mathrm{OA}} = 4\boldsymbol{i}$, $\overrightarrow{\mathrm{OC}} = 3\boldsymbol{j}$, $\overrightarrow{\mathrm{OD}} = 2\boldsymbol{k}$.



(i) Find $\overrightarrow{\mathrm{OB}}$.

(ii) Find $\overrightarrow{\mathrm{OF}}$.

(iii) Show that $\overrightarrow{\mathrm{AG}} = -4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$.

1b. *[4 marks]*

Write down a vector equation for

(i) the line OF;

(ii) the line AG.

1c. *[7 marks]*

Find the obtuse angle between the lines OF and AG.

2a. *[3 marks]*

A line L_1 passes though points P(−1, 6, −1) and Q(0, 4, 1) .

(i) Show that $\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$.

(ii) Hence, write down an equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

2b. *[7 marks]*

A second line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + s\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$.

Find the cosine of the angle between $\overrightarrow{\mathrm{PQ}}$ and L_2.

2c. *[7 marks]*

The lines L_1 and L_2 intersect at the point R. Find the coordinates of R.

3a. *[6 marks]*

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

(a) Find

(i) $2\boldsymbol{a} + \boldsymbol{b}$;

(ii) $|2\boldsymbol{a} + \boldsymbol{b}|$.

Let $2\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = \mathbf{0}$, where $\mathbf{0}$ is the zero vector.

(b) Find $\boldsymbol{c}$.

3b. *[4 marks]*

Find

(i) $2\boldsymbol{a} + \boldsymbol{b}$;

(ii) $|2\boldsymbol{a} + \boldsymbol{b}|$.

3c. *[2 marks]*

Let $2\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = \mathbf{0}$, where $\mathbf{0}$ is the zero vector.

Find $\boldsymbol{c}$.

4a. *[2 marks]*

Consider points A$(1, -2, -1)$, B$(7, -4, 3)$ and C$(1, -2, 3)$. The line L_1 passes through C and is parallel to $\overrightarrow{\mathrm{AB}}$.

Find $\overrightarrow{\mathrm{AB}}$.

4b. *[2 marks]*

Hence, write down a vector equation for L_1.

4c. *[3 marks]*

A second line, L_2, is given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 2 \\ 15 \end{pmatrix} + s \begin{pmatrix} 3 \\ -3 \\ p \end{pmatrix}$.

Given that L_1 is perpendicular to L_2, show that $p = -6$.

4d. *[7 marks]*

The line L_1 intersects the line L_2 at point Q. Find the x-coordinate of Q.

5. *[7 marks]*

Line L_1 has equation $\boldsymbol{r}_1 = \begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s \begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix}$ and line L_2 has equation $\boldsymbol{r}_2 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$.

Lines L_1 and L_2 intersect at point A. Find the coordinates of A.

6a. *[3 marks]*

Consider the points A$(5, 2, 1)$, B$(6, 5, 3)$, and C$(7, 6, a + 1)$, $a \in \mathbb{R}$.

Find

(i) $\overrightarrow{\mathrm{AB}}$;

(ii) $\overrightarrow{\mathrm{AC}}$.

6b. *[4 marks]*

Let q be the angle between $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$.

Find the value of ***a*** for which $\mathrm{q} = \frac{\pi}{2}$.

6c. *[8 marks]*

i. Show that $\cos q = \frac{2a+14}{\sqrt{14a^2+280}}$.

ii. Hence, find the value of a for which $\mathrm{q} = 1.2$.

6d. *[4 marks]*

Hence, find the value of a for which $\mathrm{q} = 1.2$.

7a. *[1 mark]*

The line L_1 passes through the points $\mathrm{A}(2, 1, 4)$ and $\mathrm{B}(1, 1, 5)$.

Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

7b. *[1 mark]*

Hence, write down a direction vector for L_1;

7c. *[2 marks]*

Hence, write down a vector equation for L_1.

7d. *[6 marks]*

Another line L_2 has equation ***r*** = $\begin{pmatrix} 4 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$. The lines L_1 and L_2 intersect at the point P.

Find the coordinates of P.

7e. *[1 mark]*

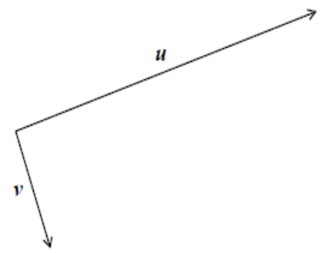
Write down a direction vector for L_2.

7f. *[6 marks]*

Hence, find the angle between L_1 and L_2.

8a. *[2 marks]*

The following diagram shows two perpendicular vectors ***u*** and ***v***.



Let $\boldsymbol{w} = \boldsymbol{u} - \boldsymbol{v}$. Represent $\boldsymbol{w}$ on the diagram above.

8b. *[4 marks]*

Given that $u = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $v = \begin{pmatrix} 5 \\ n \\ 3 \end{pmatrix}$, where $n \in \mathbb{Z}$, find \(n\).

9a. *[2 marks]*

The line L is parallel to the vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

Find the gradient of the line L.

9b. *[3 marks]*

The line L passes through the point $(9, 4)$.

Find the equation of the line L in the form $y = ax + b$.

9c. *[2 marks]*

Write down a vector equation for the line L.

10a. *[3 marks]*

Distances in this question are in metres.

Ryan and Jack have model airplanes, which take off from level ground. Jack's airplane takes off after Ryan's.

The position of Ryan's airplane t seconds after it takes off is given by $\boldsymbol{r} = \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} + t\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$.

Find the speed of Ryan's airplane.

10b. *[2 marks]*

Find the height of Ryan's airplane after two seconds.

10c. *[5 marks]*

The position of Jack's airplane $\boldsymbol{s}$ seconds after **it** takes off is given by $\boldsymbol{r}$ = $\begin{pmatrix} -39 \\ 44 \\ 0 \end{pmatrix} + \boldsymbol{s}\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$.

Show that the paths of the airplanes are perpendicular.

10d. *[5 marks]*

The two airplanes collide at the point $(-23, 20, 28)$.

How long after Ryan's airplane takes off does Jack's airplane take off?

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1a. *[5 marks]*

Markscheme

(i) valid approach ***(M1)***

e.g. $\text{OA} + \text{OB}$

$\overrightarrow{\text{OB}} = 4\boldsymbol{i} + 3\boldsymbol{j}$ ***A1 N2***

(ii) valid approach ***(M1)***

e.g. $\overrightarrow{\text{OA}}+\overrightarrow{\text{AB}}+\overrightarrow{\text{BF}}$; $\overrightarrow{\text{OB}}+\overrightarrow{\text{BF}}$; $\overrightarrow{\text{OC}}+\overrightarrow{\text{CG}}+\overrightarrow{\text{GF}}$

$\overrightarrow{\text{OF}} = 4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$ ***A1 N2***

(iii) correct approach ***A1***

e.g. $\overrightarrow{\text{AO}}+\overrightarrow{\text{OC}}+\overrightarrow{\text{CG}}$; $\overrightarrow{\text{AB}}+\overrightarrow{\text{BF}}+\overrightarrow{\text{FG}}$; $\overrightarrow{\text{AB}}+\overrightarrow{\text{BC}}+\overrightarrow{\text{CG}}$

$\overrightarrow{\text{AG}} = -4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$ ***AG N0***

[5 marks]

Examiners report

Although a large proportion of candidates managed to answer this question, their biggest challenge was the use of a proper notation to represent the vectors and vector equations of lines.

In part (a), finding $\overrightarrow{\text{OB}}$ and $\overrightarrow{\text{OF}}$ was generally well done, although many lost the mark for (iii) due to poor working or not clearly showing the result.

1b. *[4 marks]*

Markscheme

(i) any correct equation for (OF) in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ ***A2 N2***

where $\boldsymbol{a}$ is 0 or $4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$, and $\boldsymbol{b}$ is a scalar multiple of $4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$

e.g. $\boldsymbol{r} = t(4, 3, 2)$, $\boldsymbol{r} = \begin{pmatrix} 4t \\ 3t \\ 2t \end{pmatrix}$, $\boldsymbol{r} = 4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k} + t(4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k})$

(ii) any correct equation for (AG) in the form $\boldsymbol{r} = \boldsymbol{a} + s\boldsymbol{b}$ ***A2 N2***

where $\boldsymbol{a}$ is $4\boldsymbol{i}$ or $3\boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{b}$ is a scalar multiple of $-4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$

e.g. $\boldsymbol{r} = (4, 0, 0) + s(-4, 3, 2)$, $\boldsymbol{r} = \begin{pmatrix} 4-4s \\ 3s \\ 2s \end{pmatrix}$, $\boldsymbol{r} = 3\boldsymbol{j} + 2\boldsymbol{k} + s(-4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k})$

[4 marks]

Examiners report

Part (b) was very poorly done. Not all the students recognized which correct position vectors they had to use to write the equations of the lines. It was seen that they frequently failed to present the equations in the required format, which prevented these candidates from achieving full marks. The notations generally seen were $\text{AG} = \boldsymbol{a} + \boldsymbol{b}t$, $\boldsymbol{r} = 4 + t(4, 3, 2)$ or $L = \boldsymbol{a} + \boldsymbol{b}t$.

1c. *[7 marks]*

Markscheme

choosing correct direction vectors, $\overrightarrow{\mathrm{OF}}$ and $\overrightarrow{\mathrm{AG}}$ ***(A1)(A1)***

scalar product $= -16 + 9 + 4\ (= -3)$ ***(A1)***

magnitudes $\sqrt{4^2 + 3^2 + 2^2}$, $\sqrt{(-4)^2 + 3^2 + 2^2}$, $(\sqrt{29}, \sqrt{29})$ ***(A1)(A1)***

substitution into formula ***M1***

e.g. $\cos\theta = \frac{-16+9+4}{\left(\sqrt{4^2+3^2+2^2}\right)\times\sqrt{(-4)^2+3^2+2^2}} = \left(-\frac{3}{29}\right)$

95.93777°, 1.67443 radians

$\theta = 95.9^\circ$ or 1.67 ***A1 N4***

[7 marks]

Examiners report

Most achieved the correct result in part (c) with many others gaining most of the marks as follow through from choosing incorrect vectors. Some students did not state which vectors had been used, another cause for losing marks. A few showed poor notation, including ***i***, ***j*** and ***k*** in the working.

2a. *[3 marks]*

Markscheme

(i) evidence of correct approach ***A1***

e.g. $\overrightarrow{\mathrm{PQ}} = \overrightarrow{\mathrm{OQ}} - \overrightarrow{\mathrm{OP}}$, $Q - P$

$\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ ***AG N0***

(ii) any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ ***A2 N2***

where ***a*** is either $\overrightarrow{\mathrm{OP}}$ or $\overrightarrow{\mathrm{OQ}}$ and ***b*** is a scalar multiple of $\overrightarrow{\mathrm{PQ}}$

e.g. $\boldsymbol{r} = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$, $\boldsymbol{r} = \begin{pmatrix} t \\ 4-2t \\ 1+2t \end{pmatrix}$, $\boldsymbol{r} = 4\boldsymbol{j} + \boldsymbol{k} + t(\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k})$

[3 marks]

Examiners report

A pleasing number of candidates were successful on this straightforward vector and line question. Part (a) was generally well answered, although a few candidates still labelled their line $L =$ or used a position vector for the direction vector. Follow-through marking allowed full recovery from the latter error.

2b. *[7 marks]*

Markscheme

choosing a correct direction vector for L_2 ***(A1)***

e.g. $\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$

finding scalar products and magnitudes ***(A1)(A1)(A1)***

scalar product $= 1(3) - 2(0) + 2(-4)\ (= -5)$

magnitudes $= \sqrt{1^2 + (-2)^2 + 2^2}\ (= 3)$, $\sqrt{3^2 + 0^2 + (-4)^2}\ (= 5)$

substitution into formula ***M1***

e.g. $\cos\theta = \frac{-5}{\sqrt{9} \times \sqrt{25}}$

$\cos\theta = -\frac{1}{3}$ ***A2 N5***

[7 marks]

Examiners report

Few candidates wrote down their direction vector in part (b) which led to lost follow-through marks, and a common error was finding an incorrect scalar product due to difficulty multiplying by zero.

2c. *[7 marks]*

Markscheme

evidence of valid approach ***(M1)***

e.g. equating lines, $L_1 = L_2$

EITHER

one correct equation in one variable ***A2***

e.g. $6 - 2t = 2$

OR

two correct equations in two variables ***A1A1***

e.g. $2t + 4s = 0$, $t - 3s = 5$

THEN

attempt to solve ***(M1)***

one correct parameter ***A1***

e.g. $t = 2$, $s = -1$

correct substitution of either parameter ***(A1)***

e.g. $\boldsymbol{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + (-1)\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$, $\boldsymbol{r} = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} + (+2)\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

coordinates $\mathrm{R}(1,\ 2,\ 3)$ ***A1 N3***

[7 marks]

Examiners report

Part (c) was generally well understood with some candidates realizing that the equation in just one variable led to the correct parameter more quickly than solving a system of two equations to find both parameters. Some candidates gave the answer as (*s*, *t*) instead of substituting those parameters, indicating a more rote understanding of the problem. Another common error was using the same parameter for both lines.

There were an alarming number of misreads of negative signs from the question or from the candidate working.

3a. *[6 marks]*

Markscheme

(a) (i) $2\boldsymbol{a} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$ ***(A1)***

correct expression for $2\boldsymbol{a} + \boldsymbol{b}$ ***A1 N2***

eg $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$, $(5, -2)$, $5\boldsymbol{i} - 2\boldsymbol{j}$

(ii) correct substitution into length formula ***(A1)***

eg $\sqrt{5^2 + 2^2}$, $\sqrt{5^2 + -2^2}$

$|2\boldsymbol{a} + \boldsymbol{b}| = \sqrt{29}$ ***A1 N2***

[4 marks]

(b) valid approach ***(M1)***

eg $\boldsymbol{c} = -(2\boldsymbol{a} + \boldsymbol{b})$, $5 + x = 0$, $-2 + y = 0$

$\boldsymbol{c} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ ***A1 N2***

[2 marks]

3b. *[4 marks]*

Markscheme

(i) $2\boldsymbol{a} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$ ***(A1)***

correct expression for $2\boldsymbol{a} + \boldsymbol{b}$ ***A1 N2***

eg $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$, $(5, -2)$, $5\boldsymbol{i} - 2\boldsymbol{j}$

(ii) correct substitution into length formula ***(A1)***

eg $\sqrt{5^2 + 2^2}$, $\sqrt{5^2 + -2^2}$

$|2\boldsymbol{a} + \boldsymbol{b}| = \sqrt{29}$ ***A1 N2***

[4 marks]

Examiners report

Most candidates comfortably applied algebraic techniques to find new vectors.

3c. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $\boldsymbol{c} = -(2\boldsymbol{a} + \boldsymbol{b})$, $5 + x = 0$, $-2 + y = 0$

$\boldsymbol{c} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ ***A1 N2***

[2 marks]

Examiners report

Most candidates comfortably applied algebraic techniques to find new vectors. However, a significant number of candidates answered part (b) as the absolute numerical value of the vector components, which suggests a misunderstanding of the modulus notation. Those who understood the notation easily made the calculation.

4a. *[2 marks]*

Markscheme

valid approach ***(M1)***

eg $\begin{pmatrix} 7 \\ -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$, $\mathrm{A} - \mathrm{B}$, $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{AO}} + \overrightarrow{\mathrm{OB}}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$ ***A1 N2***

[2 marks]

Examiners report

While many candidates can find a vector given two points, few could write down a fully correct vector equation of a line.

4b. *[2 marks]*

Markscheme

any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\boldsymbol{b}$ is a scalar multiple of $\overrightarrow{\mathrm{AB}}$ ***A2 N2***

eg $\boldsymbol{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$, $(x, y, z) = (1, -2, 3) + t(3, -1, 2)$, $\boldsymbol{r} = \begin{pmatrix} 1 + 6t \\ -2 - 2t \\ 3 + 4t \end{pmatrix}$

Note: Award ***A1*** for $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for $L_1 = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[2 marks]

Examiners report

While many candidates can find a vector given two points, few could write down a fully correct vector equation of a line. Most candidates wrote their equation as "$L_1 =$", which misrepresents that the resulting equation must still be a vector.

4c. *[3 marks]*

Markscheme

recognizing that scalar product $= 0$ (seen anywhere) ***R1***

correct calculation of scalar product ***(A1)***

eg $6(3) - 2(-3) + 4p$, $18 + 6 + 4p$

correct working ***A1***

eg $24 + 4p = 0$, $4p = -24$

$p = -6$ ***AG N0***

[3 marks]

Examiners report

Those who recognized that vector perpendicularity means the scalar product is zero found little difficulty answering part (b). Occasionally a candidate would use the given $p = 6$ to show the scalar product is zero. However, working backward from the given answer earns no marks in a question that requires candidates to show that this value is achieved.

4d. *[7 marks]*

Markscheme

setting lines equal ***(M1)***

eg $L_1 = L_2$, $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 15 \end{pmatrix} + s\begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix}$

any two correct equations with **different** parameters ***A1A1***

eg $1 + 6t = 1 + 3s$, $-2 - 2t = 2 - 3s$, $3 + 4t = 15 - 6s$

attempt to solve **their** simultaneous equations ***(M1)***

one correct parameter ***A1***

eg $t = \frac{1}{2}$, $s = \frac{5}{3}$

attempt to substitute parameter into vector equation ***(M1)***

eg $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$, $1 + \frac{1}{2} \times 6$

$x = 4$ (accept (4, -3, 5), ignore incorrect values for y and z) ***A1 N3***

[7 marks]

Examiners report

While many candidates knew to set the lines equal to find an intersection point, a surprising number could not carry the process to correct completion. Some could not solve a simultaneous pair of equations, and for those who did, some did not know what to do with the parameter value. Another common error was to set the vector equations equal using the same parameter, from which the candidates did not recognize a system to solve. Furthermore, it is interesting to note that while only one parameter value is needed to answer the question, most candidates find or attempt to find both, presumably out of habit in the algorithm.

5. *[7 marks]*

Markscheme

appropriate approach ***(M1)***

eg $\begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$, $L_1 = L_2$

any two correct equations ***A1A1***

eg $10 + 2s = 2 + 3t$, $6 - 5s = 1 + 5t$, $-1 - 2s = -3 + 2t$

attempt to solve ***(M1)***

eg substituting one equation into another

one correct parameter ***A1***

eg $s = -1$, $t = 2$

correct substitution ***(A1)***

eg $2+3(2)$, $1+5(2)$, $-3+2(2)$

A $=(8, 11, 1)$ (accept column vector) ***A1 N4***

[7 marks]

Examiners report

Most students were able to set up one or more equations, but few chose to use their GDCs to solve the resulting system. Algebraic errors prevented many of these candidates from obtaining the final three marks. Some candidates stopped after finding the value of s and/or t.

6a. *[3 marks]*

Markscheme

(i) appropriate approach ***(M1)***

eg $\overrightarrow{\mathrm{AO}}+\overrightarrow{\mathrm{OB}}$, $\mathrm{B}-\mathrm{A}$

$\overrightarrow{\mathrm{AB}}=\begin{pmatrix}1\\3\\2\end{pmatrix}$ ***A1 N2***

(ii) $\overrightarrow{\mathrm{AC}}=\begin{pmatrix}2\\4\\a\end{pmatrix}$ ***A1 N1***

[3 marks]

Examiners report

The majority of candidates successfully found the vectors between the given points in part (a).

6b. *[4 marks]*

Markscheme

valid reasoning (seen anywhere) ***R1***

eg scalar product is zero, $\cos\frac{\pi}{2}=\frac{\boldsymbol{u}\bullet\boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}$

correct scalar product of **their** $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$ (may be seen in part (c)) ***(A1)***

eg $1(2)+3(4)+2(a)$

correct working for **their** $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$ ***(A1)***

eg $2a+14$, $2a=-14$

$a=-7$ ***A1 N3***

[4 marks]

Examiners report

In part (b), while most candidates correctly found the value of a, many unnecessarily worked with the magnitudes of the vectors, sometimes leading to algebra errors.

6c. *[8 marks]*

Markscheme

correct magnitudes (may be seen in (b)) ***(A1)(A1)***

$\sqrt{1^2+3^2+2^2}\left(=\sqrt{14}\right)$, $\sqrt{2^2+4^2+a^2}\left(=\sqrt{20+a^2}\right)$

substitution into formula ***(M1)***

eg $\cos\theta\ \frac{1\times2+3\times4+2\times a}{\sqrt{1^2+3^2+2^2}\sqrt{2^2+4^2+a^2}}$, $\frac{14+2a}{\sqrt{14}\sqrt{4+16+a^2}}$

simplification leading to required answer ***A1***

eg $\cos\theta=\frac{14+2a}{\sqrt{14}\sqrt{20+a^2}}$

$\cos\theta=\frac{2a+14}{\sqrt{14a^2+280}}$ ***AG N0***

[4 marks]

correct setup ***(A1)***

eg $\cos 1.2=\frac{2a+14}{\sqrt{14a^2+280}}$

valid attempt to solve ***(M1)***

eg sketch, $\frac{2a+14}{\sqrt{14a^2+280}}-\cos 1.2=0$, attempt to square

$a=-3.25$ ***A2 N3***

[4 marks]

Examiners report

Some candidates showed a minimum of working in part (c)(i); in a "show that" question, candidates need to ensure that their working clearly leads to the answer given. A common error was simplifying the magnitude of vector AC to $\sqrt{20a^2}$ instead of $\sqrt{20+a^2}$.

In part (c)(ii), a disappointing number of candidates embarked on a usually fruitless quest for an algebraic solution rather than simply solving the resulting equation with their GDC. Many of these candidates showed quite weak algebra manipulation skills, with errors involving the square root occurring in a myriad of ways.

6d. *[4 marks]*

Markscheme

correct setup ***(A1)***

eg $\cos 1.2=\frac{2a+14}{\sqrt{14a^2+280}}$

valid attempt to solve ***(M1)***

eg sketch, $\frac{2a+14}{\sqrt{14a^2+280}}-\cos 1.2=0$, attempt to square

$a=-3.25$ ***A2 N3***

[4 marks]

Examiners report

In part (c)(ii), a disappointing number of candidates embarked on a usually fruitless quest for an algebraic solution rather than simply solving the resulting equation with their GDC. Many of these candidates showed quite weak algebra manipulation skills, with errors involving the square root occurring in a myriad of ways.

7a. *[1 mark]*

Markscheme

correct approach ***A1***

eg $\begin{pmatrix}1\\1\\5\end{pmatrix}-\begin{pmatrix}2\\1\\4\end{pmatrix}$, $\text{AO}+\text{OB}$, $b-a$

$\overrightarrow{\text{AB}}=\begin{pmatrix}-1\\0\\1\end{pmatrix}$ ***AG N0***

[1 mark]

7b. *[1 mark]*

Markscheme

correct vector (or any multiple) ***A1 N1***

eg **d** = $\begin{pmatrix}-1\\0\\1\end{pmatrix}$

[1 mark]

7c. *[2 marks]*

Markscheme

any correct equation in the form ***r*** = ***a*** + *t**b*** (accept any parameter for *t*)

where ***a*** is $\begin{pmatrix}2\\1\\4\end{pmatrix}$ or $\begin{pmatrix}1\\1\\5\end{pmatrix}$, and ***b*** is a scalar multiple of $\begin{pmatrix}-1\\0\\1\end{pmatrix}$ ***A2 N2***

eg ***r*** = $\begin{pmatrix}1\\1\\5\end{pmatrix}+t\begin{pmatrix}-1\\0\\1\end{pmatrix}$, $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2-s\\1\\4+s\end{pmatrix}$

Note: Award ***A1*** for ***a*** + *t**b***, ***A1*** for L_1 = ***a*** + *t**b***, ***A0*** for ***r*** = ***b*** + *t**a***.

[2 marks]

7d. *[6 marks]*

Markscheme

valid approach ***(M1)***

eg $r_1=r_2$, $\begin{pmatrix}2\\1\\4\end{pmatrix}+t\begin{pmatrix}-1\\0\\1\end{pmatrix}=\begin{pmatrix}4\\7\\-4\end{pmatrix}+s\begin{pmatrix}0\\-1\\1\end{pmatrix}$

one correct equation in one parameter ***A1***

eg $2-t=4, 1=7-s, 1-t=4$

attempt to solve ***(M1)***

eg $2-4=t, s=7-1, t=1-4$

one correct parameter ***A1***

eg $t=-2, s=6, t=-3$,

attempt to substitute **their** parameter into vector equation ***(M1)*** *eg* $\begin{pmatrix} 4 \\ 7 \\ -4 \end{pmatrix} + 6 \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

P(4, 1, 2) (accept position vector) ***A1 N2***

[6 marks]

7e. *[1 mark]*

Markscheme

correct direction vector for L_2 ***A1 N1***

eg $\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}$

[1 mark]

7f. *[6 marks]*

Markscheme

correct scalar product and magnitudes for **their** direction vectors ***(A1)(A1)(A1)***

scalar product $= 0 \times -1 + -1 \times 0 + 1 \times 1 \ (= 1)$

magnitudes $= \sqrt{0^2 + (-1)^2 + 1^2},\ \sqrt{-1^2 + 0^2 + 1^2}\ (\sqrt{2},\ \sqrt{2})$

attempt to substitute **their** values into formula ***M1***

eg $\frac{0+0+1}{\left(\sqrt{0^2+(-1)^2+1^2}\right)\times\left(\sqrt{-1^2+0^2+1^2}\right)}$, $\frac{1}{\sqrt{2}\times\sqrt{2}}$

correct value for cosine, $\frac{1}{2}$ ***A1***

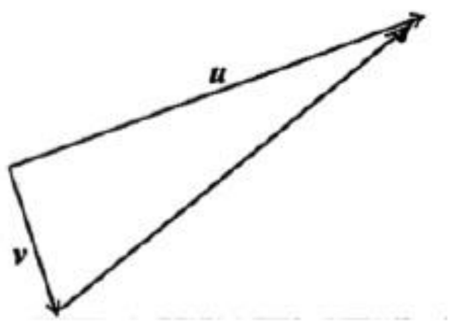
angle is $\frac{\pi}{3}$ $(= 60^\circ)$ ***A1 N1***

[6 marks]

8a. *[2 marks]*

Markscheme

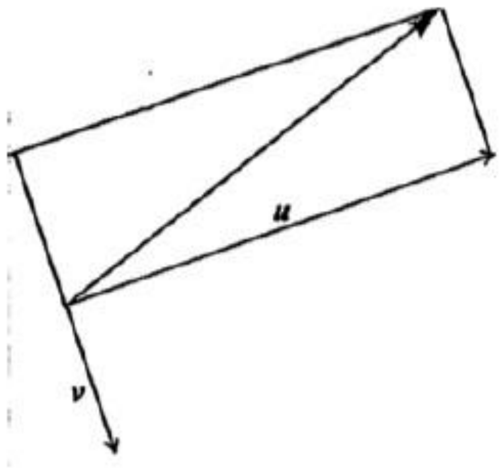
METHOD 1



A1A1 N2

Note: Award ***A1*** for segment connecting endpoints and ***A1*** for direction (must see arrow).

METHOD 2



A1A1 N2

Notes: Award ***A1*** for segment connecting endpoints and ***A1*** for direction (must see arrow).

Additional lines not required.

[2 marks]

8b. *[4 marks]*

Markscheme

evidence of setting scalar product equal to zero (seen anywhere) ***R1***

eg $\boldsymbol{u} \cdot \boldsymbol{v} = 0,\ 15 + 2n + 3 = 0$

correct expression for scalar product ***(A1)***

eg $3 \times 5 + 2 \times n + 1 \times 3,\ 2n + 18 = 0$

attempt to solve equation ***(M1)***

eg $2n = -18$

$n = -9$ ***A1 N3***

[4 marks]

9a. *[2 marks]*

Markscheme

attempt to find gradient ***(M1)***

eg reference to change in x is 3 and/or y is 2, $\frac{3}{2}$

gradient $= \frac{2}{3}$ ***A1 N2***

[2 marks]

9b. *[3 marks]*

Markscheme

attempt to substitute coordinates and/or gradient into Cartesian equation

for a line ***(M1)***

eg $y - 4 = m(x - 9),\ y = \frac{2}{3}x + b,\ 9 = a(4) + c$

correct substitution ***(A1)***

eg $4 = \frac{2}{3}(9) + c,\ y - 4 = \frac{2}{3}(x - 9)$

$y = \frac{2}{3}x - 2\ \left(\text{accept } a = \frac{2}{3},\ b = -2\right)$ ***A1 N2***

[3 marks]

9c. *[2 marks]*

Markscheme

any correct equation in the form ***r*** = ***a*** + *t****b*** (any parameter for *t*), where ***a*** indicates position *eg* $\begin{pmatrix} 9 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$, and ***b*** is a scalar multiple of $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

eg ***r*** = $\begin{pmatrix} 9 \\ 4 \end{pmatrix} + t\begin{pmatrix} 3 \\ 2 \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3t+9 \\ 2t+4 \end{pmatrix}$, ***r*** = 0*i* − 2 *j* + *s*(3*i* + 2 *j*)***A2 N2***

Note: Award ***A1*** for ***a*** + *t****b***, ***A1*** for *L* = ***a*** + *t****b***, ***A0*** for ***r*** = ***b*** + *t****a***.

[2 marks]

10a. *[3 marks]*

Markscheme

valid approach ***(M1)***

eg magnitude of direction vector

correct working ***(A1)***

eg $\sqrt{(-4)^2+2^2+4^2},\ \sqrt{-4^2+2^2+4^2}$

$6\ (\text{ms}^{-1})$ ***A1 N2***

[3 marks]

10b. *[2 marks]*

Markscheme

substituting 2 for t ***(A1)***

eg $0+2(4)$, ***r*** = $\begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} + 2\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} -3 \\ 10 \\ 8 \end{pmatrix}$, $y=10$

8 (metres) ***A1 N2***

[2 marks]

10c. *[5 marks]*

Markscheme

METHOD 1

choosing correct direction vectors $\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$ ***(A1)(A1)***

evidence of scalar product ***M1***

eg ***a*** · ***b***

correct substitution into scalar product ***(A1)***

eg $(-4\times 4)+(2\times -6)+(4\times 7)$

evidence of correct calculation of the scalar product as 0 ***A1***

eg $-16-12+28=0$

directions are perpendicular ***AG N0***

METHOD 2

choosing correct direction vectors $\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$ ***(A1)(A1)***

attempt to find angle between vectors ***M1***

correct substitution into numerator ***A1***

eg $\cos\theta = \frac{-16-12+28}{|a||b|}$, $\cos\theta = 0$

$\theta = 90^\circ$ ***A1***

directions are perpendicular ***AG N0***

[5 marks]

10d. *[5 marks]*

Markscheme

METHOD 1

one correct equation for Ryan's airplane ***(A1)***

eg $5 - 4t = -23,\ 6 + 2t = 20,\ 0 + 4t = 28$

$t = 7$ ***A1***

one correct equation for Jack's airplane ***(A1)***

eg $-39 + 4s = -23,\ 44 - 6s = 20,\ 0 + 7s = 28$

$s = 4$ ***A1***

3 (seconds later) ***A1 N2***

METHOD 2

valid approach ***(M1)***

eg $\begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} + t\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -39 \\ 44 \\ 0 \end{pmatrix} + s\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$, one correct equation

two correct equations ***(A1)***

eg $5 - 4t = -39 + 4s,\ 6 + 2t = 44 - 6s,\ 4t = 7s$

$t = 7$ ***A1***

$s = 4$ ***A1***

3 (seconds later) ***A1 N2***

[5 marks]